RIS Discussion Paper Series

SDG Targets 2.2 and 4.2 Early Childhood Nutrition and Education: Bedrock of Life-long Equity

Pramod Kumar Anand
Krishna Kumar

Discussion Paper # 287

RIS@40
RIS
Research and Information System for Developing Countries
विकासशील देशों की अनुसंधान एवं सूचना प्रणाली

SDG Targets 2.2 and 4.2
Early Childhood Nutrition and Education: Bedrock of Life-long Equity

Pramod Kumar Anand

Krishna Kumar

RIS-DP # 287

December 2023

RIS
Research and Information System for Developing Countries
विकासशील देशों की अनुसंधान एवं सूचना प्रणाली

Core IV-B, Fourth Floor, India Habitat Centre
Lodhi Road, New Delhi – 110 003 (India)
Tel: +91-11-2468 2177/2180; Fax: +91-11-2468 2173/74
Email: dgoffice@ris.org.in

SDG Targets 2.2 and 4.2
Early Childhood Nutrition and Education: Bedrock of Life-long Equity

Pramod Kumar Anand*

Krishna Kumar*

Abstract: Early childhood constitutes the formative years of life, necessitating that no inequalities build up or deepen. In this paper, the focus is on nutrition, education and household incomes being critical for fair and just early child care and development. The effect of circumstances and efforts is covered along with the interaction terms. As the Gini coefficient captures only inequality *per se*, a generic inequity augmented Lorenz curve is evolved as a contribution to literature, based on a distribution revealed parameter to ensure uniqueness. Measurement of 'within' and 'in-between' sub-group components of the Gini coefficients is extended to cover cases of 'overlaps' where a member of a disadvantaged group performs better than one or more members of the other sub-group. In this paper such members are called as 'star performers' and a 'star performance (SP) index' is also evolved as a contribution to literature. Vertical product differentiation between 'high' and 'low' quality schools is analysed. Various ways to aim at equity through affirmative actions are also analysed. Conclusions and recommendations are added to facilitate and expand policy space.

Keywords: additive decomposition of inequality, equality, equity, generic distribution revealed inequity augmented Lorenz curve, global public good, nutrition, poverty sensitising, SDGs, star performer, star performance index, vertical product differentiation

1 Introduction

Equity during early childhood has its vital effect on the life-long equity, and necessitates multiple actions across various sustainable development goals (SDGs) including SDG 2 on zero hunger and SDG 4 on quality education. The prominent targets encompassing childhood equity are 2.2

* Visiting Fellow, RIS. Email: pk.anand@ris.org.in; krishna.kumar@ris.org.in.
Authors are thankful to Prof. Sachin Chaturvedi, Director General, RIS for his guidance and valuable comments at different stages of the study. The views expressed are not of RIS but personal. Usual disclaimers apply.

and target 4.2. Target 2.2 includes to end malnutrition in all its forms, while covering intermediate targets on stunting and wasting for 2025, and nutrition related needs of adolescent girls, pregnant and lactating women. Target 4.2 aims for access to quality early child development, care and pre-primary education both for all girls and boys. In fact, nutrition and education at least up to primary level need to be made universal and free, treating childhood as a global pure public good. The UN SDG Report 2023 brings out the need for a fundamental shift to achieve the nutrition targets meant for 2030, adding on education that even if national targets are achieved millions of children would remain out of school[1]. The Tashkent Declaration and Commitments to Action for Transforming Early Childhood Care and Education (2022) on education repeatedly emphasise on SDG target 4.2 related aspects[2]. These two targets can help in ending poverty including of children, in all its dimensions, reducing mortality rates, achieving better grades, and averting forced child labour.

The challenges of childhood, namely undernutrition, low-quality school education and household's multidimensional poverty, are deeply intertwined. Any inclusive policies that promote adequate nutrition, quality education and enhance household employability leading to adequate incomes, can prove to be the *sine qua non* of early childhood care, development and progress. Eventually, these become key factors in evolving any affirmative policy actions.

In pursuit of such policy actions, it is realised that the trilemma of choice among efficiency, equality and equity remains a classical challenge. These choices being not mutually exclusive, a search for the right proportion for the policy issue at hand keeps engaged the minds of philosophers, academicians and economists alike. A narrow discouraging strand is that absolute equalities in any space, say incomes, even if once attained, cannot be continued as *ceteris paribus* some people may have backward bending labour supply curves commencing at lesser hours preferring leisure over work. Further an argument adduced, overlooking adverse circumstances faced by many persons and households, is that varying levels of efforts, wealth, as well as choice of work would put different persons on different earning trajectories.

The larger issue of tackling all kinds of malnutrition, including undernutrition component under it can't be totally overlooked, as besides the 2030 SDG targets, the intermittent 2025 targets levels on these need to be achieved in each country, for which both enhanced allocation of domestic resources and deeper international collaborations need to be facilitated.

Prevalence of triple burden of malnutrition among children encompasses three components, first undernutrition manifested as stunting, wasting and underweight; second overweight including obesity; and third micronutrient deficiency. By definition stunting is the condition of height for age (HFA) for a child under five years of age being below minus two standard deviations from the median of the WHO reference population standards for age, whereas its subset is severe stunting being below minus three standard deviations. Wasting and its subset severe wasting are similarly defined in the case of weight for height (WFH). Underweight, and its subset severe underweight, are similar manifestations related to weight for age (WFA). On the other hand, overweight and its subset obesity are indicative of weight above plus two and plus three such standard deviations respectively.

Proper nutrition and education also add to the self-respect and dignity of a person and a household, leading toward higher level of well-being. Globally the benefits from early childhood education are not limited to the current time but extend to life-long human capital build-up of the present and next generations, making it the most critical need. It also necessitates life-long learning, to upkeep with changing times, and is not limited to academics but to skilling, and upkeep with new frontiers like digital literacy.

In this paper the current section of introduction is followed by section 2 on review of literature and section 3 on challenges. Section 4 encompasses generic distribution revealed inequity augmented Lorenz curve and section 5 decomposition of the Gini coefficient and star performers. Early childhood nutrition is covered in section 6, and quality of education, school choices and vertical product differentiation in section 7. The aspects of equity aimed for and affirmative actions are covered in section 8, followed by conclusions in section 9 and finally recommendations in section 10.

2 Review of Literature

Nandy *et al.* (2005) argue[3] that undernutrition needs to be measured by the alternative 'composite index of anthropometric failure (CIAF)', which includes facing of one or more of the situations of stunting, wasting and underweight. Obviously, CIAF indicates a much higher proportion of children, than under any single failure. However, it overlooks the fact that with limited resources, to address the policy issue as to on which children to focus to reduce undernutrition, the obvious choice should be such children who are facing all the three manifestations of undernutrition i.e. stunting, underweight and wasting.

It is also felt that use of technology can help in identification of such children on real-time basis to fast track corrective interventions. Nandy *et al.* (2012) argue[4] that on the basis of NFHS 5 predictive morbidity risks of diarrhoea, dysentery and acute respiratory infection (ARI) for children under five years, can be computed by using odds ratios. They add that risk is the highest for the children falling under the category of stunting, wasting and underweight i.e. those suffering from all the three anthropometric failures. Pomati *et al.* (2019) argue[5] that there is need to consider children facing any one form of undernutrition as well as those facing more than one form. They accordingly add to consider underweight children also besides those facing stunting and wasting. Obviously leaving children facing even one of the undernutrition situations would miss on reporting of millions of children towards SDG2.

In the context of nutrition for newly born to one year old children, Maharana *et al.* argue[6] that decline in infant mortality rate (IMR) in demographically under developed states of Uttar Pradesh and Madhya Pradesh in India, assessed through multiple regression and decomposition analysis, is attributable to mother's being educated, child's birth weight (2.5 kg or more), mother's age at child birth (18 or more), use of clean cooking fuel, and being non-poor. They find that the first two factors contribute a high 79 and 19 per cent respectively of the explained difference in fall in IMR in MP; whereas 26 and 39 per cent respectively of the fall in UP. Such an analysis is indicative of efficacy of different initiatives and helps in setting an evidence-based policy roadmap. Culyer (2015) argues[7] good health as being central to individual and societal wellbeing. Chaturvedi *et al.* argue[8] that social enterprises may be effective in education and health in the context of vulnerable and marginalized sections.

On the issue of education, Ashenfelter argues[9] on the basis of a sample of 700 identical (monozygotic) twins that each additional year of schooling adds to an additional ten percent of income and that the results are slightly higher for less able students, and further adds that education and income are amenable to changes in public and private choices. Spence argues[10] that due to informational gaps, the employers offer wages signalled by attributes like education.

Amadeo argues that the cycle of inequality affects income, wealth and education of next generation. He adds that it slows down economic growth[11]. The US equity plan[12] cites educator Mann that education has the capacity to be the great equalizer, indicating that education is a generational commitment, not limited to a one-year project. Burgess *et al.* based on a study of poor and other children in the UK, argue that gap between them in bagging at least five A* to C grades was 6.9 per cent, being equivalent to a high 41 per cent of standard deviation.[13]

OECD argues that equity should cover two intertwined dimensions first, fairness and second, inclusion. It adds that fairness prohibits any discrimination, whereas inclusion is for minimum basic level of standards.[14] Castelli (2012) argues[15] that the concepts of equality and equity can be interpreted in many different ways anchoring to rights and social justice. Bendoussi (2006), analysing scholastic evolution over a number of decades, argues that in the 1960s and 1970s the concepts of efficacy and efficiency were under focus, with emphasis later shifting to quality in the 1970s and 1980s, followed by emergence of the approach of equity in the next decades. Importance of access to higher education is underscored while analysing the educational justice (Piketty 2014).[16] Piketty adds that the state of any education system also determines the supply of skills.[17]

In real life situations we find that a child born in a poor family has to earmark his time for household chores, like care of sibling and elders, and to sometimes undertake supportive livelihood activities; which reduce time available for studies compared to an average child. Felix[18] argues that in the case of two product qualities and two firms, at stage one, each firm chooses quality, at stage two, responds to its own and rival's quality and sets the price for its product. At stage three a consumer opts between the two qualities depending upon how much she cares for the better quality.

Dunaway[19] argues that comparison needs to be of prices net of marginal costs across the markets, it costs to add quality, and the cost function is increasing and convex. On school choice, Hafalir *et al.* argue that the goal of affirmative action is to give underrepresented groups higher chances to attend a better school, and that based on algorithms, reserves for such groups are a better option than fixing quotas (like ceilings) for better represented groups.[20] On the issue of admissions in medical schools, Wouters *et al.* argue that neither selection nor lottery fulfills the differing needs of applicants, medical schools and parents.[21] On the challenge of measurement of inequality in education it is argued (Antoninis *et al.* 2016)[22] that as individuals have varying abilities as well as competencies, short of expecting equality of outcomes, the circumstances at birth should not amplify differences. On inter-country comparisons, they advocate to measure completion rates by location, wealth and gender; and how temporally the different portions of concentration curve (Lorenz curve) indicate relative changes over years.

On the issue of efforts the prizes for performance are defended (Okun 1975)[23], along with recognition that circumstances being uneven starting positions, necessitate actions to make the race fairer. The leaky bucket argument (Okun 1975)[24] also indicates that only a part of taxes collected reaches as subsidies. He points out administrative costs, adverse effects on the economic incentives (for rich and poor), besides advocating for a progressive income tax.[25]

While some of the interventions need supporting fiscal space, the resultant gains, even from the narrow lens of fiscal balancing need to be internalised. It is a fact that reduction in prevalence of undernutrition, and for that matter of overweight / obesity, can help in higher life-long productivity, resulting in a higher net present value of the stream of additional taxes pouring into the exchequer. In fact, in the short run too, timely arrival of an ambulance for an expecting mother can avert huge hospitalisation costs, not only for the child but also for her. With an emphasis on the promotion of technological progress, Boushey (2017)[26] argues that policies need to support equitable distribution of the resultant bounties/ benefits. Shields argues[27] that equality of opportunity related to social goods differs in the case of education, as first, it has a central place and myriad opportunities; second, high quality educational opportunities are scarce for many children; and third, the state has a critical role in providing opportunities in education.

Recognising the importance of the Gini coefficient, Lorenzo Giovanni Bellù *et al.* argue[28] that Gini coefficient is the most popular inequality index. On the issue of additive decomposition of the Gini index, Bellù *et al.* in another paper further argue[29] that in case of any overlap in ranks between sub-groups, a term needs to be added to use the Gini index for the decomposition of inequality. Arguing about the meaning of this term, they express that it is not very intuitive. In this paper, we would delve deeper to bring forth how this term can be computed and utilised, in evolving a new index by giving an algebraic methodology.

Fairlie (1999)[30] argues that for non-linear equations, the decomposition analysis can be based on the use of cumulative distribution functions of logistic distribution.

3 Challenges

3.1 Inequality and Inequity

The most critical challenge to early childhood policies is how to set up the aspired normative objective being looked for. Whereas equality in any space say, not being underweight for given age, or on student scores or household income, is relatively easier to conceive, but equity throws up many questions as to equity of what, especially given that nutrition and education shape adult life outcomes including health and incomes. For a person in the lowest decile of income, equity entails that her adverse circumstances be duly compensated for to avert undernutrition and attain education, whereas her efforts be fully rewarded.

A person's efforts made and adverse/ favorable circumstances faced have a large bearing on her achievements. To conceive a policy on equity, the major challenge is to disentangle effects of circumstances from that of efforts. A practical challenge in evolving any policy toward it is to ascertain the contribution of circumstances and efforts, along with of the interaction term between circumstances and efforts, i.e. the effect of circumstances on efforts and the effect of efforts on circumstances. For instance, a primary school child from a poor household may be facing circumstances like stunting or being entrusted with the care of a sibling or studying in a dilapidated school all leading to deceleration of efforts. Similarly, the efforts of a child may fetch her a scholarship mitigating the effect of adverse circumstances. To overcome the responsibility of taking care of a sibling at the cost of own studies, a crèche facility at the place of work of the mother can help out. On the challenge of who should facilitate resources to overcome the adverse effects of circumstances, it is expected that the State provides the same. On a long-term horizon, a more equitous treatment enhances dignity and competence, which are paid back to the society as better productivity, and even from the narrow financial angle in the form of the likely stream of additional tax revenues.

Worse circumstances include being a disabled or orphan or homeless child; whereas facing more than one such adversity makes childhood even harsher. Such a wide range of circumstances faced by different children weaken their potential efforts in varying manners, rendering the choice of a set of redressing policy prescriptions much more difficult. Moreover, by nature time being an exhaustible resource over each child's lifetime, the requisite time for means-testing of the children and their families, delays redressal and worsens circumstances, among which unfortunately effects like stunting may become totally irreversible. All the more, the time lived with mental trauma is worth avoiding, for which societally acceptable minimal standards of living should be in place, at least for all such persons who are ready to make efforts commensurate with their capabilities, and further improve upon.

On the deeper challenge of minimising inequalities in health Jusot *et al.* examine[31] three alternative specifications of inequalities drawing on Roemer, Barry and Swift's considerations of circumstances, effort and interaction term of circumstances with effort. They argue that as per Barry's view[32], the correlation is split between effort and circumstances according to the rules of regression, under Roemer's view[33], all the correlation is treated as a circumstance, whereas under the Swift's view[34], all the correlation adds to effort. They argue that the last two views are extreme, and can only be used to assess lower and upper bounds of the contribution of effort to reduce inequality. The challenge of interactions makes it difficult to analyse equality of opportunity, for example, on the aspect of health, initiation of smoking is found to be negatively related to mother's education, and positively related to parents' smoking behaviour, thus internalization of efforts of parents also become relevant.

To overcome the adverse circumstances, admission of a child from a 'poor' family in a better quality school remains a major qualitative challenge having life-long implications. Amadeo argues that equality in education entails that each student attains an acceptable level. He further argues that the gap due to such a school choice accounts for a high 37 per cent lag in mathematics scores.[35]

In this paper the challenge of school choice, i.e. choice of school to study in, as faced by the students and parents is covered at length while assessing the 'in-between' inequality on scores, and building up the differentiation in the quality of education. On nutrition, the efforts are not limited to adequately earning in order to afford food. These include opting for nutrition-rich food, and overcoming the challenges of becoming obese or becoming a smoker. Further, better handling of stress can help in being calm, balanced and thus more productive for the society.

Sen[36] argues on 'equality of what?', and mentions that plurality of space and diversity of individuals need to be taken note of. He further argues that there are different ways of evaluating equality in the same homogeneous space, using distinct methods of measuring inequality, like coefficient of variation, Gini coefficient, standard deviation of logarithms, measures of entropy. Arneson[37] argues that what Barry presents is an egalitarian theory of social justice, built on the doctrine of equality of opportunity. He adds this doctrine to hold that all people should be made equal or close to equal. In the context of Sen's argument on 'equality of what and in which space', Arneson further underscores equality being in the spaces of rights, opportunities, and resources, but not due to well-informed voluntary choices.

In the light of the preceding thoughts, apparently, in the battle between choice of equality and equity, the challenge is further muddled as equity in a way aims to restore the missed equality and averts a fresh one to build up. It is not only important that any critical intervention is continued for the targeted people, it is equally important that its value shouldn't be stagnated in nominal terms, thereby allowing it to diminish in real terms. Besley *et al.* argue[38] that in case the real value of any food stamp intervention is allowed to fall it can lead to severe consequences for poverty and intervention. Moreover, coverage needs to be flexible to add new households that become eligible and to de-duplicate and weed out those that are false, or of persons permanently migrated, died or no longer eligible. For instance, in India 42.8 million bogus ration cards were cancelled under the targeted public distribution system (TPDS), with the use of IT for reforms and digitisation of data, over almost seven years ending 2021.[39]

3.2 Challenge of Measurement

In the debate over equity and equality, opting in favour of equality may not suffice, if equality aims at only the equality of opportunity. This leaves the gap owing to the initial inequalities uncovered. A striking example is of the children with special needs (CWSN), who are bound to lag behind if the pedagogical methods evolved in general are mechanically applied to them. In this paper our focus is on equality of outcomes, which entails more than equality of opportunity. In the context of education, a 'poor' household may aspire for their child getting just enough score to 'pass' an exam, whereas a 'rich' household may aspire for their child to get a distinction. In this paper, the terms 'poor' and 'rich' are confined to being below arithmetic mean (AM) of income or otherwise. Therefore, from the legacy of adverse circumstances sub-optimal outcomes also need to be purged out, in favour of choice for equal outcomes and similar efforts needed to achieve so, once the education ecosystem compensates for adverse circumstances.

In this paper, our focus would be to minimise the 'in-between' sub-group differences. As a strategy to attain it, firstly, in the context of income, prescription may internalise taxing the 'rich' to 'subsidise' the 'poor'. Secondly, in the context of education outcomes, affirmative action to improve the scores of each student in the lagging sub-group, at least up to the threshold score of say, 50 per cent, but without discouraging any child from a 'rich' household in any manner. Thirdly, in the case of nutrition, say, to increase the weight of children below five years, without breaching the ceiling of median plus two standard deviations of the reference population, thus not becoming overweight.

A number of indices, including Theil-T (or T_T) and Theil-L (or T_L) are used to measure inequality. These two indices are in fact special cases of the general entropy index E (α), which is expressed as:

$$E(\alpha) = [1/\{N^*(\alpha^2 - \alpha)\}] * [\sum \{(x_i / \bar{x})^\alpha - 1\}]$$

Here, the subscript 'i' varies from 1 to N, and $\bar{x}$ is the notation for the arithmetic mean (AM). Now, as the parameter alpha approaches 0 or 1, both the numerator and denominator tend to zero throwing up indeterminate forms. Therefore, the application of L'Hopital's Rule is resorted to in conjunction with the Taylor's expansion, and in each case the first derivative is found to make the ratio determinate.[40]

When alpha equals zero, the ratio is called Theil-L and expressed as:

$E(0) = [1/(N)] * \sum [\ln (\bar{x} / x_i)]$ (the summation being over all the 'i' values).

Where, $\bar{x}$ = AM (arithmetic mean). In fact, Theil-L index is anchored to GM (geometric mean) also besides AM, as it can be expressed as the ln of the N^{th} root of $\{(AM)^N / (GM)^N\}$ or ln(AM/GM), its genesis emanating from the fact that when any inequality exists, AM exceeds GM, so their ratio exceeds unity and its logarithm (on any base) is invariably positive.

Or, on tossing the argument of 'ln' and thus putting the negative sign outside, it is alternatively expressed as:

$E(0) = [-1/(N)]*\sum \ln (x_i / \bar{x})]$ and as the expression suggests, it is the Mean Log Deviation (MLD), comprised of the average value of ($\ln x_i - \ln \bar{x}$), albeit with a negative sign. When alpha equals one, the ratio is called Theil-T index and can be expressed as:

$E(1) = [1/(N)]*\sum [(x_i / \bar{x}) * \{\ln (x_i / \bar{x})\}]$ (the summation being over all the N values). Therefore, when all terms are equal, the value of Theil-T index becomes zero. In case income of a household tends to zero from above, its contribution (inside the bracket), tends to $(x_i / AM) * \{\ln (x_i / AM)\}]$, which being indeterminate of the form (zero*infinity), having a negative sign, can be written as {y*ln(y)} or [{ln(y)}/ {1/y}], where y= (x_i/AM). On separate first differentiation of numerator and denominator, it gives $\{(1/y) / (-1/y^2)\}$ or (-) y, which is now determinate as it tends to zero. Therefore, in the case of extreme inequality, the contribution inside the bracket of each of the (N-1) 'poor' households tends to zero, whereas of the sole 'rich' having an income of N*AM becomes (N*AM/ AM)*ln(N*AM/AM) or N*ln(N), resulting into maximum value of the Theil-T index as ln(N), after multiplication by the (1/N) appearing outside the bracket.

Next, on inclusion of the concept of entropy of a distribution to capture information, in the case of extreme inequality as one household has the entire income, it leaves no uncertainty and thus entropy is zero. As the distribution changes towards randomisation, entropy keeps on increasing towards ln(N).

The Theil-T index can also be expressed as T = ln(N) –S. Here, S is the Shannon entropy or information content of the distribution (also computed by using ln the natural logarithm), rising from zero for extreme inequality, to ln(N) for equality of all incomes, resulting into Theil-T falling from ln(N) to zero respectively. Moreover, for a given distribution the Lorenz curve and the related value of Theil-T index are also interlinked (Rhode, Nicholas 2007).

If the given terms can be partitioned into disjointed sub-groups in a mutually exclusive and collectively exhaustive (MECE) manner, the Theil-T index has the property of being additively decomposed[41] as its expression is the sum of the following two terms.

The first term, called the sum of 'within' terms equals:

$[1/(N)]*[\sum \sum [(x_i / \bar{x}_j)\{\ln (x_i / \bar{x}_j)\}]$, the inner summation being within each sub-group 'j' using the sub-group arithmetic mean $\bar{x}_j$ followed by the outer summation across all the sub-groups j = 1, 2,3, … p.

The second term, called the 'in-between' term equals:

$[1/(N)]*[\sum [(\bar{x}_j / \bar{x})\{\ln (\bar{x}_j / \bar{x})\}]$, for computing it each value of a sub-group 'p' is thus first replaced by the arithmetic mean $\bar{x}_j$ of the sub-group.

A major shortcoming of Theil-T index is that by construction it attaches higher weightage to bigger values, for instance, in the case of monthly incomes of three households, being 7, 13 and 19 (thousand INRs) and thus averaging 13, the three contributing terms before division by three are: (7/13)*{ln(7/13)}, (13/13)*{ln(13/13)} and (19/13)*{ln(19/13)}, or (-) 0.3333, 0 and 0.5546, adding up to 0.2213 yielding on division by three the value of Theil-T index as 0.0738. Notably, the contribution of the term below AM is negative, and though the first and the third terms are equidistant from the AM, the higher term contributes more in absolute value too, and this leads to a positive total value.

By contrast, Theil-L index is defined as $[1/(N)]*\sum\{\ln (\bar{x}_i / x_i)\}$. The contributions of the three terms in this example are {ln(13/7)}, {ln(13/13)} and ln(13/19)}, or 0.6190, 0 and (-) 0.3795, adding up to 0.2395 yielding on division by three the value of Theil-L index as 0.0798, which happens to be slightly higher than the value of Theil-T index of these three terms. Notably, towards this index, the term below AM makes a positive contribution, which for it is mathematically higher in absolute terms, than from an equidistant term above AM. However, a shortcoming of the Theil-L index is that it is open ended, as even for one term being very small compared to AM, its value tends to infinity.

4 Generic Distribution Revealed Inequity Augmented Lorenz Curve

4.1 A Fair Tax Subsidy Rule

A theoretical SDG target 10.1 experiment (Anand and Kumar, 2022)[42] can be carried out over say, 100 households with incomes (per period) uniformly increasing from 100 to 1,000 (units), manifesting a 10:1 ratio between the highest and the lowest incomes. Such a distribution has an Arithmetic Mean (AM) of 550, which occurs between the incomes of 50th and 51st households. The Gini coefficient of this distribution is computed to be 0.275, indicating that the Lorenz curve would show an area of the lens as 27.5 per cent of the triangle below the egalitarian line. This is a balanced distribution in the sense that for it the mean and the median precisely coincide.

A related concept is a fair tax/ subsidy rule using a parameter 't', the letter 't' being indicative of a proportional tax/ subsidy rate on a term's distance to AM. Thus, under it, on each 'rich' defined as having income above AM, a tax rate 't' is imposed on the portion of the income exceeding AM. Similarly, for each 'poor' defined as having income below AM, a subsidy is given on the shortfall from AM. Therefore, such proportional tax/ subsidy rate is pivoted to AM.

In general, algebraically for any distribution if its N terms are x_1, x_2, x_3, … x_N, leading to Arithmetic mean as AM, the tax and subsidy rule conceived is that for x_i > AM, a tax of $t*(x_i - AM)$ is imposed, and for (x_i < AM) a subsidy of $t*(AM-x_i)$ is granted, which is a negative tax of $t*(x_i - AM)$ being the same expression again algebraically, where $0 \leq t \leq 1$ (i.e. $0 \leq 100$ per cent).

Thus, net receipt to the exchequer being the sum of total taxes minus total subsidies, can be computed as,

$\Sigma\, t*(x_i - AM)$ over all the x_i from 1, 2 to N, which is essentially nothing but,

$t*(N*AM - N*AM)$ or zero. Therefore, it is a fair tax/ subsidy rule distributing all the taxes collected from 'rich', as subsidies among 'poor'.

The new arithmetic mean becomes (1/N) * Σ $\{x_i + t*(x_i - AM)\}$, or (1/N) {N*AM + 0}, or AM, the same as the initial AM.

Accordingly, this tax/ subsidy rule is also an AM preserving operation. Further, as t < 1, any two initial incomes x_g and x_h, where $x_g < x_h$ on tax/ subsidy application become $\{x_g - t*(x_g - AM)\}$ and $\{x_h - t*(x_h - AM)\}$, and the new difference between these (second minus first) becomes $\{(x_h - x_g) - t*(x_h - x_g)\}$, or $(1-t)*(x_h - x_g)$ which is positive as t< 1 and $x_h > x_g$. Therefore, the income ranks are also preserved, on application of this tax/ subsidy rule. Moreover, the tax is progressive in nature, as it increases with income.

4.2 The Gini Coefficient: *ab initio* computation

It is known by definition that that the Gini coefficient in essence captures (normalised) double summation of absolute differences, without assigning any extra weightage to terms below AM compared to the terms above AM. Therefore, it is an indicator of inequality *per se*, but not of inequity. It is a fact that multiplication (or division) of each term by the same factor leaves the value of the Gini coefficient unchanged, as for normalisation the double summation of absolute differences needs to be divided by the new AM which also becomes doubled, while the remaining divisor term $2*N^2$ remains unchanged**.** In other words one can say that the Gini coefficient follows the relative income principle, besides of course the anonymity principle, population (cloning) principle and the Pigou-Dalton transfer principle.

In this paper, to have a feel of its distributional inequity a generic endogenised distribution revealed parameter 'r', capturing an element of inequity is computed. This generic parameter can be expressed as the sum of all terms below the median, divided by the sum of all terms above the median, while for any term/s exactly equal to the median assigning half each of its value to the numerator and the denominator. If the number of terms is even, to start with all terms are arranged in the ascending order and median computed as the arithmetic mean of the two middle terms. Next, the lower of the two middle terms needs to be added to the numerator, while the higher one to the denominator, being below and above median respectively.

Therefore, algebraically the generic distribution revealed inequity parameter is expressible as r = (Bg/ Ag) where,

Bg = (sum of terms below median + half of any term/s equal to median)

Ag: (sum of terms above median + half of any term/s equal to median)

As by definition, the number of terms below and above the median is equal, the ratio 'r' is invariably less than or equal to one, the case of equality occurring only when each term is equal, rendering no inequality. Incidentally, in the special case of the SDG 10.1 experiment mentioned in this section, the AM and the median being precisely identical could be interchangeably used.

From the ratio 'r', which is unique for a given distribution, one can compute a parameter 's' say, called the inequity gap parameter defined as (1-r), which is used as the distribution revealed endogenisd value of 't' the tax and subsidy rate for reduction of inequity under the fair tax and subsidy rule. We use the term x_i for initial values, x_{ip} for its value after the progressive tax/subsidy and x_{ia} for the augmented value, covered next.

The ratio of the post-tax/ subsidy value of each term to its initial value i.e. $(x_{ip}/x_i) = v_i$ is computed for each x_i (the subscript 'p' is to indicate progressive), thus yielding x_i specific v_i values where each $v_i \leq 1$. While the tax/ subsidy rule brings the initial x_i terms closer, by contrast division of the x_i values by the corresponding v_i values, stretches these to what we call the generic augmented values which can be expressed as $x_{ia} = (x_i/ v_i)$, or $\{x_i\}* /[\{x_i - t*(AM-x_i)\}/\{x_i\}]$ or, $[\{x_i\}^2 */\{x_i - t*(AM-x_i)\}]$. Incidentally, the geometric mean of x_{ip} and x_{ia} by design happens to be x_i. Notably, as we keep selecting distributions with increasing inequity, the parameter 't' also increases, resultantly further expanding the generic inequity augmented Lorenz curves. The extreme inequality among N terms can occur when each of the (N-1) 'poor' households earns zero income (lives on dis-savings if any, social assistance, borrowings, and the little assets) and the sole 'rich' earns the entire N*AM. Thus in the double summation, the numerator includes an (N-1)*(N-1) matrix of zeros, along with the self-pair of 'rich' contributing zero, whereas the 2*(N-1) cross pairs contribute 2*(N-1)*{N*AM}, aggregating 2*(N-1)*N*AM. Therefore, the Gini coefficient becomes [{2*(N-1)*N*AM}/ {2*N*N*AM}] or

{(N-1)/N}, which is short of 'unity' by (1/N). Thus, in the case of extreme inequality, as the number of terms rises, the Gini coefficient tends towards 'one' from below. On augmentation the value of the Gini coefficient is further pushed towards unity.

Now, for an egalitarian society as each term, the median and the AM are identical, r is 1, so s is zero needing no tax or subsidy operation, each v_i value is 1, and values of x_i, x_{ip} and x_{ia} are identical. Thus, the generic augmented Lorenz curve coincides with the original egalitarian line, devoid of any element of inequality or revealed inequity.

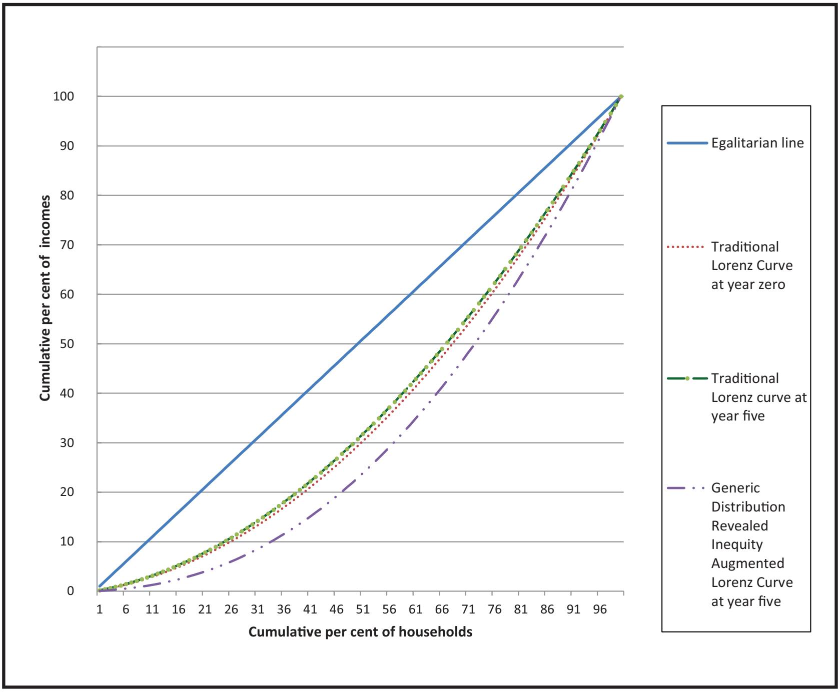
In a nutshell, the Lorenz curve of the N values of x_{ia} leads to what is termed in this Paper as the 'Generic Distribution Revealed Inequity Augmented Lorenz Curve'. These operations are essential for differential multiplicative augmentations, as by contrast uniform multiplication by any positive real number leaves the value of the Gini coefficient unchanged.

We next carry out five-year convergent income growths. For this we select, say an annual growth rate of six per cent for the lowest income, with growth rates tapering down uniformly for higher incomes and falling to three per cent for the highest income. Next as a depiction of the SDG 10.1 experiment, we draw the egalitarian line and the following three curves as in **Figure 1**:

i. Lorenz curve for the original distribution of incomes in year zero
ii. Traditional Lorenz curve after five-year convergence of incomes
iii. Generic distribution revealed inequity augmented Lorenz curve at year five corresponding to the curve at (ii), which is unique for a given distribution as evolved in this paper

One finds that even after five-year annual differential growth rates, the traditional Lorenz curve in this experiment moves only a little towards the egalitarian line. Therefore, more robust affirmative policy changes are necessitated to achieve substantial equity. The level of inequity is still high after five years as captured in the large augmentation of the post five-year innermost curve into the outermost curve.

Figure 1 : Traditional and Generic Distribution Revealed Inequity Augmented Lorenz Curves- with five-year income Convergence



Source: Authors' own depiction.

5. Decomposition of the Gini Coefficient and Star Performers

5.1 The Gini Coefficient *ab initio*

The Gini coefficient for N values comprising of pairs like x_i and x_j having a positive arithmetic mean as AM is *ab initio* computed from the double summation of absolute differences as:

$$G = \{ \Sigma \Sigma \,|\, x_i - x_j \,|\, \} / \{AM*(2*N^2) \}$$

The literature extends to cover an analysis of negative terms also resulting into the Gini coefficient falling outside the closed interval zero to one, and using correcting factors (Battisti *et al.* 2019)[43]. However, in this paper we confine to the mainstream, using positive terms (or zeros but with at least one positive term), and thus a positive AM. In the case of continuous values, of course a double integration needs to be carried out to compute the Gini coefficient.

Some properties of the Gini coefficient and Lorenz curve are covered next.

Firstly, while each Lorenz curve has a unique Gini coefficient, its reverse is not true; as intersecting lenses of Lorenz curves may have equal non-overlapping areas in two differing distributions.

Secondly, in case all the terms are equal, and thus the resultant AM is also so, each absolute difference is zero, whereas the denominator being 2*AM*N*N or non-zero; renders the Gini coefficient as zero. In fact, the corresponding Lorenz curve is represented by the egalitarian line itself.

Thirdly, in the case of the extreme inequality, with each of the (N-1) 'poor' earning zero, and the sole 'rich' earning N*AM, the 'within' sum of 'poor' from (N-1)*(N-1) matrix, and 'within' 'rich' one term contribute zero to double summation. The 'in-between' 2*(N-1) terms contribute 2*(N-1)*(N*AM), and as already shown the Gini coefficient becomes {(N-1)/N}, which is short of 'unity' by (1/N).

Fourthly, if each term is multiplied by a positive scalar say, ζ (zeta), the resultant double summation becomes ζ-fold, and the new arithmetic mean becomes ζ*AM, preserving the value of the Gini coefficient. This in turn shows that this index holds the relative income principle.

Fifthly, the addition of a positive scalar say Θ (capital theta), to each term, leads to the i^{th} term to become $(x_i + \Theta)$, whereas the first term (lowest in value) becomes $\{(x_1 + \Theta)/x_1\}$. If we now carry out a division of each new term by $(x_1 + \Theta)$, to revert the first term to x_1, the operation shrinks the i^{th} term to $x_1 * \{(x_i + \Theta) / \{(x_1 + \Theta)\}$ which is less than x_i. With the new terms being relatively closer, the Gini coefficient falls. On the other hand subtraction by Θ, retaining each term positive (i.e. $\Theta < x_1$), enhances the value of the Gini coefficient.

Sixthly, any AM preserving Pigou-Dalton regressive transfer between two households, one 'poor' earning 'a' to one 'rich' earning 'c' (where a < AM < c), making these as (a-k) and (c+k), shrinks the Lorenz curve reducing the value of the Gini coefficient. Algebraically, as the (N-2) terms are left unchanged the intra (N-2)*(N-2) matrix of absolute differences remains unchanged. Obviously, the contributions of self-pairs of (a-k) and (c+k) yield zeros, as did self-pairs of 'a' and 'c'. Let us assume that the value of the transfer 'k' being small, doesn't change any rank. Now let n_l be the number of terms lower than 'a', and n_h the number of terms higher than c. The fall in 'a' leads to an increase by 'k' in $(N-n_l)$ terms along with decrease by 'k' in n_l terms, resulting in a net increase of $(N-2*n_l)*k$ in the sum of absolute differences. Further, the rise in 'c' by 'k' leads to an increase by 'k' in $(N-n_h)$ terms and decrease by 'k' in n_h terms, leading to $(N-2*n_h)*k$ as an increase in the sum of absolute differences. In totality, the increase in the sum of absolute differences is thus $(2*N-2*n_l -2*n_h)*k$, or $2*(N-n_l-n_h)*k$, which is positive as $(n_l + n_k)$ is invariably below N by definition (maximum sum of these numbers would still fall short of N by two).

As an aside, if fall by the value 'k' brings 'a' below another term, the decrease in contribution to double summation, would be lower being 'k' each for $(n_l -1)$ terms and less than 'k' for one such other term, in place of $(n_l)*k$, whereas increases remaining unchanged, leading to rather higher inequality. Similarly, if increase by value 'k' brings 'c' above another term, the fall would be lower being 'k' each for $(n_h -1)$ terms and less than 'k' for one such other term, whereas increases remaining unchanged, thus leading to higher inequality.

Therefore, such a regressive transfer increases value of the Gini coefficient. The idea of using a regressive transfer is merely technical so that on its application no term switches across AM, as normatively size of any transfer should not make a 'poor' as 'rich' at the cost of a 'rich' becoming 'poor', is also inherent in the progressive fair tax and subsidy rule, which is rank preserving (as only a rate of 't' below 100 per cent is socio-economically tenable).

Seventhly, the point of tangency to the Lorenz curve also yields average income in AM terms of the population percentile at that point. As such, at the population percentile pertaining to the tangent slope of unity, the average income is one percent of the total income i.e. equal to AM. Further, if in the case of distribution A the slope of unity is at the 63rd population percentile, whereas for distribution B it is at the 71st population percentile, the latter distribution has higher inequality in the portion from origin to the point of the tangent slope rising to unity. Similarly, for the slope equal to half, the point of tangency is at the percentile having half of the average income i.e. at AM/2.

Eighthly, the slope of the secant joining the origin to the end of the 40th percentile helps in finding the percentile at which the tangent is parallel to this secant. Resultantly, the slope of the tangent at such percentile on the Lorenz curve indicates the average income of the bottom 40 percent, specifically indicated in the SDG target 10.1.

Ninthly, the absolute difference $|x_i - x_j|$ between any two terms x_i (< AM) and x_j (> AM) on the different sides of AM, can be visualized as the sum of absolute differences of these terms from AM i.e. $|AM-x_i|$ + $|x_j-AM|$. Further, for the points on the same side of AM, the absolute difference can be expressed as the absolute value of the difference between the two absolute differences $|AM-x_i|$ and $|x_j-AM|$.

Tenthly, the application of the fair tax and subsidy rule shrinks all these absolute difference terms by 't' per cent, resulting in the fall of each building block of the absolute difference of the Gini coefficient by a proportion of 't', and therefore, reduces the Gini coefficient by such proportion. Thus the application of say, t = ¼ , for the fair tax and subsidy rule on the distance to AM, reduces the Gini coefficient by ¼ or 25 per cent.

5.2 Additive Decomposability of Indices

One needs to recall here how *ab initio* the Gini coefficient is computed by taking double summation over all values normalised by the Arithmetic

Mean (AM). In a dynamic sample with significant efforts across all percentiles, the education scores would be increasing and therefore pulling up their AM. As such, if the normative policy interventions are effective enough the Gini coefficient should generally diminish. However, if children of those from among 'rich' can improve their scores relatively better, the 'in-between' component of Gini coefficient would increase, along with a rising average performance of both groups. Besides, the interventions being normative, there is one more reason pointing towards the likelihood of a fall in the Gini coefficient. This is the fact that the scores are not open ended, and there is a cap akin to 100 per cent or 'A plus grade' which by design no one can exceed.

Additive decomposability of an inequality index is considered a desirable feature facilitating further granular analysis. However, for its overall utility, some other desirable features can be listed as, it being equitous in nature, collapsing to zero when all terms are identical, and ranging from zero to one.

In general, the value of an inequality index computed as say, I_G, for the group of the population having N terms, can also be computed for its mutually exclusive and collectively exhaustive (MECE) sub-groups, disaggregated by say, location (rural or urban), or incomes ('rich' or 'poor') or gender and so on.

For an index to be additively decomposable, the sum of sub-group index values i.e. 'within', and of between the sub-group values i.e. the 'in-between', should yield the value of the index for the entire group,

$$I_G = (I_1 + I_2 \ldots\ldots I_N) + I_B.$$

Here the N 'within' terms are in brackets and the 'in-between' term I_B outside. To compute I_B, all terms within each sub-group are replaced by the corresponding sub-group arithmetic mean thus collapsing the new 'within' terms to zero. When there are only two sub-groups there is only one I_B value; but in the case of n sub-groups there are ${}^{n}C_2$ or [{n*(n-1)}/2] 'in-between' values. Each such value would be the sum of 2*i*j terms, for the varying 'i' and 'j' terms, in each subgroup pair.

Literature indicates that additive decomposability based on sub-groups, holds in the cases including Theil-T index, Theil-L index (by first converting each $\{\ln(AM/x_i)\}$ term into $\{- \ln(x_i/AM)\}$, Atkinson Index; for the non-overlap case of the Gini coefficient. In this paper we would decompose the Gini coefficient for the case of overlap also.

Shorrocks[44] argues that for an index to be additively decomposable, it should be continuous and symmetric over the terms, and have a first order partial derivative which is also continuous. In line with this paper, using AM for the arithmetic mean, N for the total number of varying x_i terms, and using c as by Shorrocks, a generic additively decomposable inequality index argued by him is:

$Ic = (1/N)* [1/\{c*(c-1)\}] * [\Sigma \{(x_i/ AM)^c - 1\}]$, for c being neither 0 nor 1,

$I_0 = (1/N)*\{\Sigma \ln(AM/x_i)\}$, for c = 0 and

$Ic = (1/N)* \{\Sigma (x_i /AM)*\ln(x_i /AM)\}$, for c = 1

Now, one additively decomposable index is Theil T index, which as above for the case of c = 1 is expressed as:

$(1/N)*\sum [(x_i/AM)*\{\ln(x_i/AM)\}]$, but rather than being poverty sensitising it not only accords higher weightage to 'rich' (say, for $x_i>AM$), but also renders the contribution of each 'poor' (thus for $x_i<AM$), as negative. Theil-L index as above for the case of c = 0, being $(1/N)*\sum\{\ln(AM/ x_i)\}$, is of course, poverty sensitising, but open ended giving very high values for very 'poor', resulting into its upper value being very high even for one x_i approaching zero, causing its value to tend to infinity.

Some poverty sensitising (PS) indices are evolved in a Paper[45] covering SDG target 10.1 (Anand and Kumar), which are:

$PS1 = (1/N)\,\Sigma\{(AM-x_i) / (AM+x_i)\}$, (its variant with absolute values of each contribution called as PS1B);

$PS2 = (1/N) [\Sigma\{(AM-x_i) /(AM+x_i)\}^2]$, (and its variant on taking its positive square root called as PS2B); and

$PS3 = (1/N) [\Sigma \{(AM/x_i)*\ln (AM/x_i)\}]$.

Further, the methodology in the preceding section gives another such index called PS4, leading to the Generic Distribution Revealed Inequity Augmented Lorenz Curve, as illustrated in Figure 1. Later, in this paper we evolve PS5 naming it the 'Star Performance Index'.

In the case of the Atkinson index, the basic idea used is that the marginal utility of money is diminishing and assumed to remain positive[46], which renders the utility function as concave towards income axis. This leads to the average utility of two (or more) persons having different incomes, being less than the utility from their average income. Thus an equally distributed equivalent income, yields the same total utility for identical incomes that are lower than the original average income.

The Atkinson index can be expressed as, $[1- \{(x_{ede})/ AM\}]$ on applying the notations used in this paper. Here, x_{ede} is the 'equally distributed equivalent' income, and the parameter Ɛ indicates the level of inequality aversion as:

$x_{ede} = [\{(1/N)*\Sigma\, x_i^{(1-\varepsilon)}\}^{\{1/(1-\varepsilon)\}}]$ for $0 \leq \varepsilon \neq 1$

whereas, $x_{ede} = [\{\Pi\, (x_i)\}^{(1/N)}]$ for $\varepsilon = 1$

This index can be made poverty sensitising by using an appropriately higher choice of the level of inequality aversion. However, the value of this parameter is not revealed by the given distribution, and therefore left to choice, which leads to varying interpretations and thus to its differing values.

Importantly, in this Paper, we use the Generic Inequity Augmented Lorenz curve, for which the distribution reveals the PS sensitising parameter used. In fact, in the literature generalized Gini coefficient (Yitzhaki 1983) is also available[47], but that too necessitates first setting a specified degree of inequality aversion. But the possibility of choice of degree of aversion leads to its non-unique (in fact, innumerable) versions. Underscoring the need for a uniquely determined index, in this section we would next propound another distribution revealed unique choice of index called star performance index.

5.3 Additive Decomposability of Non-Overlapping Gini Coefficient

Let there be 'N' households (or individuals) falling into two mutually exclusive and collectively exhaustive (MECE) sub-groups, say, rural and urban. Let the 'p' rural households have incomes expressible in cardinal terms arranged in the ascending order as x_{1R}, x_{2R}, x_{3R}, …x_{pR} and their mean income as AM_R; and the 'q' i.e. (N-p) urban households having incomes arranged in the ascending order as x_{1U}, x_{2U}, x_{3U} …. x_{qU} and their mean income as AM_U.

To start with we may assume that the average urban household income is higher than (or equal to) the average rural household income $AM_U \geq AM_R$. The reverse situation (say, the average rural household being better than an average urban household; on a parameter like the quantity of fresh vegetables and fruits accessible) can be similarly handled, by an interchange of these nominal labels. Initially, we would also assume that each urban value is higher than each rural value, i.e. the lowest urban value > the highest rural value. Later on, this assumption would be relaxed.

Now, as a first step, the Gini coefficient 'within' the sub-group of rural households say, $Gini_R$ can be computed from the double summation as,

$\{ \Sigma \Sigma | x_{iR} - x_{jR} | \} / \{AM_R*(2*p^2) \}$ …..(1),

where the double summation is over all the p*p rural household income pairs and the division is for normalization. Calculation of the Gini coefficient for given values of terms can be carried out by using a software, one such being the Wessa free calculator[48].

Similarly, as a second step, the Gini coefficient 'within' the sub-group of urban households say, $Gini_U$ can be computed as,

$\{ \Sigma \Sigma | x_{iU} - x_{jU} | \} / \{AM_U*(2*q^2) \}$…..(2)

As a third step (assuming there are only two sub-groups), the literature gives a starting point on computing the Gini coefficient of the 'in-between' component, by initial replacement of each sub-group income by its sub-group mean. So here each urban income is now expressed as

AM_U and each rural income by AM_R. This yields the 2*p*q such 'in-between' i.e. the cross pair differences, post such replacements. The double summation over these 2*p*q reduced cross pairs (whereas all p*p and q*q 'within' pairs of this reduced N*N matrix collapse to zero), is therefore, $\{2*p*q*(AM_U - AM_R)\}$.

The resultant 'in-between' contribution towards the Gini coefficient (we would later see the need for a multiplicative factor of unity/ more for it to give $Gini_{between}$ when the highest x_{iR} is/ is not < the lowest x_{jU}),

$$= \{2*p*q*(AM_U - AM_R)\} / (AM*2*N*N) \quad ----(3)$$

Compared to it, the complete contribution of absolute double summation of cross terms to the Gini coefficient is,

$$= \Sigma \Sigma \mid x_{jU} - x_{iR} \mid \} / \{(AM*2*N*N)\} \quad -----(4)$$

Now, if the highest rural household income ≤ the lowest urban household income, i.e. a case of no overlap, the multiplicative factor is unity, and the equation (3) itself yields a value identical to the value given by equation (4), latter being anchored to the definitional value of 'in-between' component of the Gini coefficient, as it emanates from the absolute differences. However, in case there is an overlap, the values start diverging, necessitating the multiplication of the value given by equation (3) by more than unity to reach the true value as given by (4).

In this paper, we would also recognise the effort of any member of the sub-group facing adverse circumstances, but still performing better than even one member of the sub-group enjoying better circumstances, by assigning her the status of a '**Star performer**', so long as the parameter under analysis has a positive marginal utility. We would analyse cases of positive marginal utility like higher household income, a girl child weight higher than that of a boy at age five, so long as her not being overweight, and higher examination scores even though constrained by the upper bound of 100 per cent. The overall standard deviation of the entire group (encompassing all the sub-groups) would fall too – a desirable change.

Revisiting our analysis on rural-urban divide, now in case one rural income exceeds one urban income i.e. $(x_{iR} - x_{jU}) = \psi$ i.e. psi being positive, the rural household is a star performer. The contribution of this pair to the numerator of the cross absolute difference in double summation is $2*\psi$ or $2*(x_{iR} - x_{jU})$ ----(5). Here, one of the two terms comes from a cell in the north east (NE) rectangle of the N*N matrix, i.e. from the first to p^{th} rows, and $(p+1)^{th}$ to $(p+q)^{th}$ columns. The second term comes from the south west (SW) rectangle, i.e. from the $(p+1)^{th}$ to $(p+q)^{th}$ rows, and the first to p^{th} columns. Notably, no element of the 1^{st} to p^{th} rows and columns comprising of the p*p 'within' rural matrix, and the $(p+1)^{th}$ to $(p+q)^{th}$ rows and columns comprising of the q*q 'within' urban matrix, is used for this purpose.

Visualisation of the N*N matrix displays a 'Star Performer' like a scintillating star as an 'in-between' term, that appears twice in the double summation matrix, leading to a difference of $2*(x_{iR} - x_{jU})$, as shown in **Table 1** by marking an asterisk on such absolute differences.

In the case of an overlap, the contribution of a star performer term to the numerator of equation (3) happens to be:

$2*[(X_{jU} - X_{iR})]$ or $2*\{-(X_{iR} - X_{jU})]$ -----(4) a negative term. However, the sum of all the 2*p*q pairs becomes $2*p*q*\{\sum\sum(X_{jU} - X_{iR})\}$, or $2*p*q*\{(AM_U - AM_R)\}$ remaining positive (unless there are sufficient star performances tilting the balance, a possibility which is handled later).

Therefore, the effect of the overlap on the double summation is:

$(-)\ 2*\{(x_{iR} - x_{jU})]$ ----(5).

Further, against its positive absolute value in equation (4), the difference now being negative instead of positive, its impact in totality is its twice i.e. $2*2*(x_{iR} - x_{jU})$, or $4*\psi$, where the single overlap (NE or SW) is ψ. The same would hold for more than one star performer.

5.4 Star Performance (SP) Index the Poverty Sensitising Index 5 (PS5)

As an example covering overlaps elaborated in **Table 2,** let 5 (p) rural households have incomes as 14, 18, 31, 37 and 40 (units) leading to AM_R as 28. Further, let 4 (q) urban households have incomes 24, 36, 39 and 49 leading to AM_U as 37. We call such rural households, that have one or more income overlaps as ‘Star Performers’, and call value of the sum of such overlaps as ‘Star Performances’. This leads to three ‘Star Performer’ rural households, first with income as 40 (over 24, 36 and 39), second as 37 (over 24 and 36) and third as 31 (over 24), resulting in their respective overlap pairs indicated in Table 2 with asterisk, having respective sum of values as 2*21, 2*14 and 2*7 i.e. 42, 28 and 14 adding to 84 i.e. the total ‘Star Performance’. Therefore, the total difference due to overlaps, after accounting for equation (3) having a negative contribution compared to equation (4) having positive a contribution, gives an impact of 2*84 or 168.

Coming back to the sum of NE and SW overlap terms adding up as 84, out of the double summation of ‘in-between’ terms being 528, here in general we define the Poverty Sensitising Index 5 or PS5 naming it the ‘Star Performance Index’ or SP Index, as the ratio of:

Numerator: sum of overlap terms among the ‘in-between’ double summation absolute terms

Denominator: sum of all ‘in-between’ double summation absolute terms

or, SP Index = $(OV_{in\text{-}between}) / (ALL_{in\text{-}between})$ ------(6)

Therefore, in this case, the SP Index is (84/ 528) or (7/ 44) or 0.159. The range of the SP index would start from zero, as the ratio of sums of two absolute terms, both being positive, cannot be negative. The minimum value would pertain to the case of no overlap or no ‘star performance’, like the lowest urban income exceeding the highest rural income. The upper limit of the SP index can be unity, when say, the lowest rural income exceeds the highest urban income, and all NE and SW terms can be indicated with an asterisk. On visualization the index is in fact, sum of NE and SW cross terms indicated by an asterisk, divided by all cross terms, and therefore becomes unity. In fact, if the rural incomes keep on increasing, thus in equation (6) similar to the increasing numerator, the denominator would also keep on increasing, keeping the ratio intact at unity. Therefore, the range of the index is a very desirable zero to unity.

The SP index gives an idea of how the members of the ‘disadvantaged’ sub group are catching up on the members of the ‘advantaged’ sub-group, and their proportion in the double cross i.e. ‘in-between’ summation.

Table 1: Star Performances and ‘Within’ and ‘In-Between Double Summations of Absolute Differences

		Rural					Urban						
											Sum of ‘Within’ absolute differences		Sum of ‘ In- Between’ cross absolute differences
	Incomes										rural	urban	
		14	18	31	37	40	24	36	39	49			
Rural	14	0	4	17	23	26	10	22	25	35	70		92
	18	4	0	13	19	22	6	18	21	31	58		76
	31	17	13	0	6	9	7*	5	8	18	45		38
	37	23	19	6	0	3	13*	1*	2	12	51		28
	40	26	22	9	3	0	16*	4*	1*	9	60		30
Urban	24	10	6	7*	13*	16*	0	12	15	25		52	52
	36	22	18	5	1*	4*	12	0	3	13		28	50
	39	25	21	8	2	1*	15	3	0	10		28	57
	49	35	31	18	12	9	25	13	10	0		48	105
											284	156	528

Source: Authors’ own computations.

Here, the ‘in-between’ total absolute double summation is 528, which on truncation by 168 being the impact of overlaps, leads to the truncated sum as 360, and as expected 2*p*q*(AM_U- AM_R), or 2*5*4*(37-28) or 2*5*4*9, is also 360.

Next, in **Table 2** we may have a look at the 'within' and 'in-between' contributions to the components of the Gini coefficient, for the distribution in this example.

Table 2: Example of Equivalence of Gini Coefficient Computed from Double Summation of Absolute Differences and Contributions from 'Within' each sub-group and 'In-Between' sub-groups

		Method 1 by computing double summation of absolute differences	**Method 2 from 'Within' each sub-group and 'In-Between' sub-groups components**	
	(a)	**(b)**	**(c)**	**(d)**
		Contribution to Gini Coefficient from absolute differences	**Calculated 'Within' each sub-group and 'In-Between' sub-groups Gini coefficients needing further multiplications**	**Computed multiplicative factor for column (c) to become equal to column (b)**
1.	'Within' 5 Rural households	∑∑ Absolute differences over 5*5 terms/ {32*(2*9*9)} or, (284)/ (32*2*9*9) or, 0.0548	Over 5 rural households is ∑∑ Absolute differences over 5*5 terms / {(28*2*5*5)} or (284)/ (28*2*5*5) or, 0.2029	(5/9)*{(5*28/ 9*32)} i.e. (population share of rural) * (income share of rural) or, 0.2701
2.	'Within' 4 Urban households	∑∑ Absolute differences over 4*4 terms/ (32*2*9*9) or, (156) / (32*2*9*9) or, 0.0301	Among 4 urban households ∑∑ Absolute differences over 4*4 terms / (37* 2*4*4) or, 156/ (37*2*4*4) or, 0.1318	(4/9)*{(4*37/ 9*32)} i.e. (population share of urban) * (income share of urban) or, 0.2284

Continued...

Continued...

3.	'Between' the rural and urban households (a case of overlaps)	∑∑ Absolute differences over 2*5*4 terms/ (32*2*9*9) or, 528/ (32*2*9*9*) or, 0.1019	Being the case of overlap: {(sum excluding overlaps)} / (32*2*9*9) or, 360/ (32*2*9*9*) or 0.0694	(sum excluding overlaps + sum of overlaps) / (sum excluding overlaps) or, (360 + 168)/ 360 or, (528/ 360) or, 1.4667
		1, 2 and 3 give Gini Coefficient as (968) / (5,184) or 0.1867		

Source: Authors' own computations.

One can now move towards generalization, as in **Table 3.**

Table 3: In General Equivalence of Gini Coefficient Computed from Double Summation of Absolute Differences and Contributions from 'Within' each sub-group and 'In-Between' sub-groups

		Method 1 by computing double summation of absolute differences	**Method 2 from 'Within' each sub-group and 'In-Between' sub-groups components**	
	(a)	**(b)**	**(c)**	**(d)**
		Contribution to Gini Coefficient from absolute differences	**Calculated Gini coefficients 'Within' each sub-group needing further multiplications**	**Computed multiplicative factor for column (c) to become equal to (b)**
1.	'Within' p Rural households	∑∑ Absolute differences over p*p terms/ (AM*2*N*N)	Over p rural households is ∑∑ Absolute differences over p*p terms / {AM_R* 2*p*p)	(p/N)*{(p*AM_R)/ (N*AM)} i.e. population share of rural * income share of rural

Continued...

Continued...

2.	'Within' q Urban households	∑∑ Absolute differences over q*q terms/ (AM*2*N*N)	Among q urban households ∑∑ Absolute differences over q*q terms / (AM_U* 2*q*q)	(q/N)* {(q*AM_U/ N*AM)} i.e. population share of urban * income share of urban
3A.	**Case 1:** 'Between' the rural and urban households **for no overlaps**	∑∑ Absolute differences over 2*p*q terms/ (AM*2*N*N)	2*p*q* (AM_U -AM_R) / {(AM*2*N*N*)}	One
3B.	**Case 2:** 'Between' the rural and urban households **for overlaps**	As in case 1 being same as: {(sum excluding overlaps) + (sum of overlaps)} / (AM*2*N*N)	{(sum excluding overlaps)} / (AM*2*N*N*)	{(sum excluding overlaps) + (sum of overlaps)} / (sum excluding overlaps) This would thus be more than one in case of any overlap

Notes

1A: The expression 2*p*q* (AM_U -AM_R) / {(AM*2*N*N*)}, as given by any software for the 'in-between' sub-groups Gini component is based on the Truncated sum of cross terms arrived at by subtracting impact of overlaps from the absolute sum of cross terms. This truncated expression can also be written as:
2*{(p*q*AM_U – q*p*AM_R)} / {(AM*2*N*N*)}
or, 2* {p* income share of urban – q* income share of rural} / {2*N}
or, {(population share of rural)* (income share of urban)} – {(population share of urban)* (income share of rural)}
1B. In case of overlap this needs to be multiplied with: {(sum excluding overlaps) + (sum of overlaps)} / (sum excluding overlaps)
1C. Multiplicative factors are not 'weights' in the sense that sum of these weights is not one.
2. If $AM_U < AM_R$, one can alternatively *ab initio* use (AM_R -AM_U)

Source: Authors' own composition.

6 Early Childhood Nutrition

The aim of SDG 2 can be stated to be four-fold first, to achieve zero hunger; second, to achieve food security; third, to improve nutrition; and fourth, promotion of sustainable agriculture as the driver of the first three.

Target 2.1 of the Goal has a focus on the poor and people in vulnerable situations (in fact, substantially intersecting) and recognises the importance of infants by a specific mention in this target. Further, on target 2.2 pertaining to malnutrition, two of its three global indicators specifically mention children under five years of age facing stunting and underweight; the third being on anaemia among women of 15 to 49 years (productive age) disaggregated by pregnancy status.

We next look at the Indian Academy of Pediatrics (IAP) recommendations for the Indian children at 5-year age, after removal of data of children with weight for height exceeding two Z scores and application of Lambda, Mu and Sigma (LMS) method, while recommending the WHO Growth charts for children under age of five.[49] The IAP recommendations for age five include median weight and SD for girls as 16.4 kg. and 2.5 kg., and for boys as 17.1 kg and 3.2 kg. Separately, the gender differential in WHO Global multi-growth centre standards is also analysed which gives median weight and SD for girls as 18.2 kg and, 2.7 kg respectively, and for boys as 18.3 kg and 2.5 kg respectively (indicating in this case lower variability for boys than girls), at age five; whereas these standards are termed as the aspirational models in the IAP study. Without the loss of much generality, one can assume that these distributions are normal. However, in real life situations the weights for a given age are not exactly normally distributed and have non-zero skewness and a kurtosis other than 3 (as a normal distribution has a kurtosis of 3). For these real life distributions techniques like the LMS method can be utilised, where these letters denote Lambda, Mu and Sigma (LMS), to modify a distribution transforming it closer to the normal distribution. Here, lambda is the Box-Cox power by which the variable is raised to give the distribution closest to normality, mu is the mean and sigma is the coefficient of variation (standard deviation/

AM). This approach evolved by Cole and Green[50], allows construction of smooth reference centile curves, which can be fitted as the cubic spline curves to such a Box–Cox transformation.

Reverting to the assumed normal distributions for boys and girls at say, age five; the difference between these weights can also be treated as a new normal distribution, with its mean as the difference between the two means (medians used), and its variance as the sum of variances. Here, we are further assuming that weight of a randomly selected five-year boy is independent of the weight of a randomly selected five-year girl, and thus the variance sum law holds without need for any modification. This law states that the variance of the sum or difference of two variables, having a normal distribution, is the sum of their variances, i.e. $\sigma^2_{(x\pm y)} = \sigma^2_x + \sigma^2_y$ in which the LHS takes two positions i.e. (x+y) and (x-y), however, the RHS remains unchanged, assuming that the two variables are not correlated. In case these are dependent with a mutual coefficient of correlation of say, 'r' the equation changes to

$$\sigma^2_{(x \pm y)} = \sigma^2_x + \sigma^2_y \pm r^*\sigma_x{}^*\sigma_y$$

This implies that, in the case of a positive correlation, the variance of the sum is higher than that of the difference.

Next, in **Table 4** the data from Indian Academy of Pediatrics is analysed for assessing gender differentiation, indicating 50.57 per cent of the value of Gini coefficient being 'in-between' girls and boys, which on inequity augmentation raises the Gini coefficient by 23.5 per cent, whereas the Star Performance Index is 36.1 per cent. Separately, for the WHO Multi-growth Centre Standards a similar analysis indicates half the value of Gini coefficient being 'in-between' girls and boys, which on inequity augmentation raises the Gini coefficient by 20.5 per cent, whereas the Star Performance Index is 44.0 per cent.

Table 4: Gender Differentiation of Weights at Age of Five: Decomposition of Gini Coefficient, its Generic Distribution Revealed Inequity Augmentation and Star Performance Index Separately for IAP data and WHO Reference Population Standards

	IAP – Indian Children	Per cent of Total Gini	WHO – Reference Population	Per cent of Total Gini	Remarks
N	100		100		
p	50		50		
q	50		50		
AM	16.75		18.25		
AM_{Girls}	16.40		18.20		
SD_{Girls}	2.5		2.7		
AM_{Boys}	17.10		18.30		
SD_{Boys}	3.2		2.5		
Gini Within Girls	0.0209	21.68	0.0207	25.94	
Gini Within Boys	0.0267	27.75	0.0192	24.06	
Gini coefficient 'in-between' Girls and Boys	0.0487	50.57	0.0399	50.00	Falls absolutely but rises as a per cent
Total Gini coefficient	0.0963	100.00	0.0798	100.00	
SP (Star Performance) Index as per cent of double cross terms (after dropping the overlaps by overweight)		0.361 or 36.1 per cent	0.440 or 44.0 per cent		
r_{wts}	0.7606		r'_{wts} 0.7969		
s_{wts}	0.2394		s'_{wts} 0.2031		
Gini coefficient of the Generic Distribution Revealed Inequity Augmented Lorenz Curve	0.119 up by 23.5 per cent		0.096 up by 20.5 per cent		

Source: Authors' own computations.

7 Quality of Education School Choices and Vertical Product Differentiation

7.1 Towards Equality of Outcomes

The outcome variable of an education year can be measured in various ways; one such common choice being the scores achieved. If it is assumed that the assessment of scores is quite comprehensive and credible, the demand for equity would necessitate moving towards equality of scores, implying the 'poor' overcoming the adversity attributable to circumstances. Here, in no way we are endorsing test scores to be the gold standard, but still, the scores achieved physical standards and proficiency, the institute's reputation and ranking, subjects opted, as well as skills acquired do matter in employability; though moral attributes cannot be overlooked. However, moral attributes remain difficult to measure, and by contrast easier to fake in structured short-term exploratory interactions.

The issue of school choice for disabled students called Children With Special Needs (CWSN) remains highly critical and necessitates priortisation in any actions towards equity. Shields analyses the issue of educating such children with students without disabilities and points out that a counterview is that such children may be bullied.[51] However, we feel that for such students attending education in the same classroom is essential for their mainstreaming. An additional modality suggested in this paper is that if an institute is headed by a disabled person selected on academic merits, it may help.

In this backdrop, it is aimed that scores of students from 'poor' families studying in say, government schools, and from 'rich' families studying in private schools both increase whereas the gap between their arithmetic means diminishes. Assuming both the distributions to be normal, prior to as well as post the positive interventions, statistically in the case of the two populations the aim should be to reduce the gap between the means to zero. Further, in the case of samples, the aim should be to reduce the difference between sample means to (zero ± t*SE), the choice of the value of 't'-stat being contingent upon the level of significance opted for, with the variance sum law operating. In reality quality would vary across schools and many government schools may be having a quality better than many private schools. Stills, for simplicity of analysis, we assume that a differentiating line can be drawn between two levels of quality- low and high.

Now, a situation may crop up in which the scores of students from 'poor' families, as well as from 'rich' families both increase, but the gap between the means also increases, manifesting a case of increased inequality and in this case being against equity. Such a case may pertain to scores, but not to incomes where tax and subsidy may be used as leveling policy instruments. As such, here the policy cannot be to discourage any child and thus should be to permit each child to avail the positive intervention provided, like extra classes or interactive teaching methods.

Therefore, the objective function can be set as to minimise (100- AM_{LowQ}) or even to minimize (100- s_{min}), where, s_{min} is the minimum score, which in a dynamic setting would be across the scores of different students at different times. Reverting to (100- AM_{LowQ}), if due to an intervention, children from high quality schools improve their scores so much so that the (AM_{HighQ} –AM_{LowQ}) rather rises, it should be socially acceptable, as a possibility for the objective function selected, so long as s_{min} is rising. In a nutshell, while policy intervention may aim at addressing adverse circumstances, still no one should be discouraged to shun away from extra efforts on an attribute like education.

Most likely as the scores have an upper bound of 100 per cent, the gap should diminish in general, still if only 'rich' students are repeatedly able to gain more, the intervention itself needs to be modified to make it more equitous, and in turn reflected so in the outcomes.

7.2 Vertically Differentiated Markets for Education

Stylized facts:

i. Both in developed and developing countries the market for education is vertically differentiated, that too falling into two broad bands of quality. In the developing countries, besides many high quality private schools, a large number of state-run schools function to cater to the vast number of students from poor and vulnerable families. In developed countries to cater to students who face racial discrimination or are ineligible under the minimal residential distance criterion for high quality schools located in rich neighborhoods, low quality schools also function. There is a need to invest more in education, technology as well as infrastructure, which with more security would help in efficiency and offering more opportunities (Stiglitz 2012).[52] From the equity perspective Ulbrich argues that education is sometimes classified as a merit good, adding that it is an entitlement for being member of a society even if one is unable to pay for it.[53]

ii. In the developing and developed countries alike, State provides education for those who can't afford it at all, in some countries in the form of universal education and among them in a few as the right to education. In its quest to universalize education on the ground, its quality gets diluted. A striking feature is that compared to state-run public schools, many private schools entrust each teacher with a higher number of students, exert to provide better quality, and still manage to pay a lower compensation, partly taking advantage of the prevalent higher level of unemployment, especially among educated youth shunning away from blue-collar jobs, and partly by not insisting on minimal training stipulated/ desired for teaching.

iii. Although private schools operate in the same cities, where state-run schools impart almost free education, existential threat leaves the former with no option but to ensure a better or at least perceived better quality in order to charge a fee- mostly hefty, to run the schools.

iv. Vertical product differentiation by quality between the low and high quality schools is manifested by indicators like better pass percentage, higher average marks, higher number of first divisions or better grades, higher number of distinctions, and selections in district/ sub-national merit lists. Some relevant non-academic indicators are higher representations in sports and extra-curricular activities, education fairs etc. Incidentally, some other expenses topped up on fee by high quality schools are higher transportation costs, and

AC facilities in school halls/ classes. Nevertheless, the quality of education is maintained at a higher level to justify higher fees.

v. Parents try that their child doesn't miss out on the best in education, and may thus borrow to fructify their aspirations to get education in a better quality school. Seen from the lens of one such household, the choice between availing education and foregoing it is rooted in the concept of opportunity cost. In the literature, opportunity cost is the foregone 'second best' option while opting for the best one. For a family availing education, doing so is the best option. However, for a family opting to forego education, schooling is revealed as the second best option, the best one being retaining the child at home, maybe to take care of siblings or elders or to supplement household income through some economic contribution like assisting an artisan parent. It is essential to expand the scope of literature to see the opportunity cost from the lens of society also, to which universal education is essential. For a society, to ensure education for each child is the best option, whereas letting a household forgo it, is a much worse option. All the more a society understands that any family opting for not availing it, is merely applying an inordinately high discount rate to future gains from education. This necessitates society to frame and support policies to fructify the option of schooling.

vi. In the case of school choice, parents and a child are stuck in a unique educational dilemma and forced to eventually opt between schools offering two substantially distinct levels of quality. The **Table 5** elaborates domain related ingredients of a traditional general vertical product differentiation, and in the context of education for the specific case of school choice between high and low quality schools. The high quality schools keep the availability of seats limited to enhance profits, but in turn, causing a sub-optimal social choice. For the traditional vertical product differentiation, Machado[54] argues that in the setting of game theory, backward induction captures price led choice, assuming the existence of an indifferent consumer between the two possible quality choices, in the quality-utility space.

Table 5: Education Market: Duality under Vertical Product Differentiation

	Domain	Traditional Vertical Product Differentiation	Select salient features of the education market under analysis
1.	Product Differentiation	On a real basis like quality, defining characteristics etc.; or a fancied basis like name, colour or packaging. Products are not homogeneous, but close substitutes, and backed up by a barrier to imitation.	A high quality (Q_{high}) monopolistic range of schools (trying to enhance their market power) and a range of broadly homogenous low quality (Q_{low}) schools. Still some children don't attend any school as the household can't afford to forego the child's contribution towards care of a sibling, or the child being orphan has to earn for living
2.	Firm Entry and Exit	In the long-term entry and exit lead to zero economic profits.	By contrast high quality schools exercise monopolistic power and raise barriers against entry

Continued...

Continued...

3.	Consumers	Consumers value the 'Product differentiation' but differently, as consumer 1 rates each quality lower compared to consumer 2: i. $V^1_{high} > V^1_{low}$ ii. $V^2_{high} > V^2_{low}$ iii. $V^2_{high} > V^1_{high}$ iv. $V^2_{low} > V^1_{low}$ Consumers value high quality over the low by a margin larger than respective marginal costs v. $(V^1_{high} - V^1_{low}) > (MC_{high} - MC_{low})$ vi. $(V^2_{high} - V^2_{low}) > (MC_{high} - MC_{low})$ Consumer surplus: vii. Consumer 1 has no consumer surplus $(V^1_{low} - p_{low}) = 0$ viii. Consumer 2 has consumer surplus of $(V^2_{high} - p_{high})$ ix. Each consumer type has a participatory constraint and an incentive compatibility constraint, x. Consumer 1's participatory constraint is $(V^1_{low} - p_{low} \geq 0$, (in it when he doesn't have any consumer surplus, the sign becomes 'equal to')	(a) many 'poor' households may be valuing high quality even more than as valued by many 'rich' households. Therefore, $V^2_{high} > V^1_{high}$ may not always hold. (b) some 'poor' households may not be able to pay even zero price for Q_{low}. (c) incentive compatibility may not occur, due to inability to identify an indifferent consumer (d) market prices can not be computed

Continued...

Continued...

		Consumer 2's participatory constraint is $(V^2_{high} - p_{high}) \geq 0$ Consumer 1's incentive compatibility constraint to opt 'low' against 'high' is $(V^1_{low} - p_{low}) > (V^1_{high} - p_{high})$ Consumer 2's incentive compatibility constraint to opt 'high' against 'low' is $(V^2_{high} - p_{high}) > (V^2_{low} - p_{low})$ participatory equality of 1 and existence of incentive compatibility of the indifferent consumer 2, can lead to computation of both prices.	
4.	Demand curves	Downward sloping. Each consumer demands either one or zero unit, most of them are eager to first get the high quality,	Downward sloping, relatively inelastic for Q_{high}. Relatively elastic for Q_{low} (low price variation, while costs are largely borne by the State), and for some demand falls to zero, even for zero price. No demand by the most deprived section, out of school children (OOSC), even for zero price, therefore, price being negative a subsidy is sought

Continued...

Continued...

5.	Supply curves	Supply of Q_{high} is relatively inelastic in the short-term and kept limited in the long-term akin to a monopolistic competition. Supply of Q_{low} is elastic.	Supply of Q_{high} is relatively inelastic in the short-term and kept limited in the long-term akin to a monopolistic competition. Supply of Q_{low} is highly elastic, in line with state policy to promote education.
6.	Price (Fee) and cost	i. $P_{high} > P_{low}$ ii. $MC_{high} > MC_{low}$	The producer of Q_{high} is a price maker; whereas of Q_{low} a price taker which may be for an administered nominal price. For Q_{high} even relatively poor consumer is prepared to pay the high fee, if not possible out of her income, by dis-savings, borrowing or disposing off an asset. Price for Q_{low} is low and may be close to nil or even negative (subsidy in the forms of books, uniform, a free meal, low value education coupons).

Continued...

Continued...

7.	Profits	Marginal costs (MCs) differ. But each MC may remain within a narrow band. However, prices highly differ. Profits are positive and substantial for high quality, whereas rarely so may be even negative for low quality. In general $(P_{high} - MC_{high}) > (P_{low} - MC_{low})$ to continue price discrimination adjusted for marginal costs. In real life situations some firms may maximise profits by offering both qualities.	Teachers' compensation is kept lower for Q_{high} to maximise profits and to extract services amidst prevailing unemployment levels. For Q_{low} teachers' compensation is generally relatively higher and above the statutory minimal standards as State needs to be a model employer. In real life situations some schools may maximise profits by offering both qualities.

Source: Authors' composition.

vii. Contrary to traditional product differentiation, the scramble for high quality schools is so much that in this context there is no indifferent consumer by choice, though many are forced to opt for low quality, as the high one is missed out due to factors like wealth, neighbourhood, income, status, connections, and limited lottery.

7.3 Education Market Demand and Supply for Dual Quality

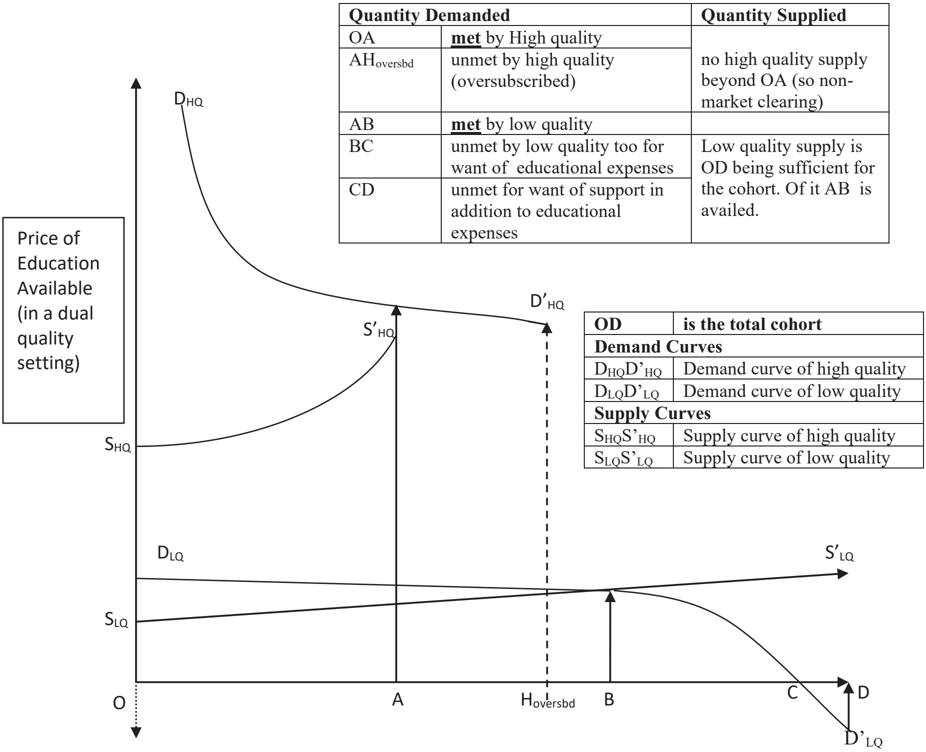
In the typical dual product differentiation setting **Figure 2** manifests the demand and supply for early childhood education based on markets for high and low quality schooling. Notably, for the high quality, as the supply happens to be or kept limited to OA, being short of the demand, it is a non-market clearing situation. Moreover, the unmet demand is carried over as a component of demand for the low quality schools.

Now children represented by AB avail low quality education. However, for the children represented by BC even the low quality education is not availed by the household, as they deeply discount the future benefits in favour of the current situation forcing them to keep the child back home. The children represented by CD are in a worse situation as they can not avail of education even when provided free of cost, and rather want subsidies to attend a low quality school, maybe to set-off the little earnings or assistance to a farmer or artisan parent, as well as due to non-economic factors like need to take care of a sibling or an elder.

Any policy framework aiming to ensure that 'no one is left behind' needs to locate, motivate and cover the children represented by BC and CD, treating childhood as a global pure public good.

From a life-long perspective **Figure 3** depicts the cumulative utility function for a household member against her cumulative education, by using years of schooling and in college as a proxy for education. The cumulative utility function is akin to $U(nQ_i, p_n)$, where n is the number of years of schooling, Q_i is the related quality like high or low (Q_{high} or Q_{low}) and p_n is the cumulative price paid for acquiring it. The aggregations for arriving at the cumulative values may be perceived as the net present values (NPVs) of the related parameters. The utility graphs may be assumed as net of prices paid. The lowest graph is a bit

Figure 2: Early Childhood Education: Market Demand and Supply for Dual Quality Vertical Differentiation for a Cohort of Children



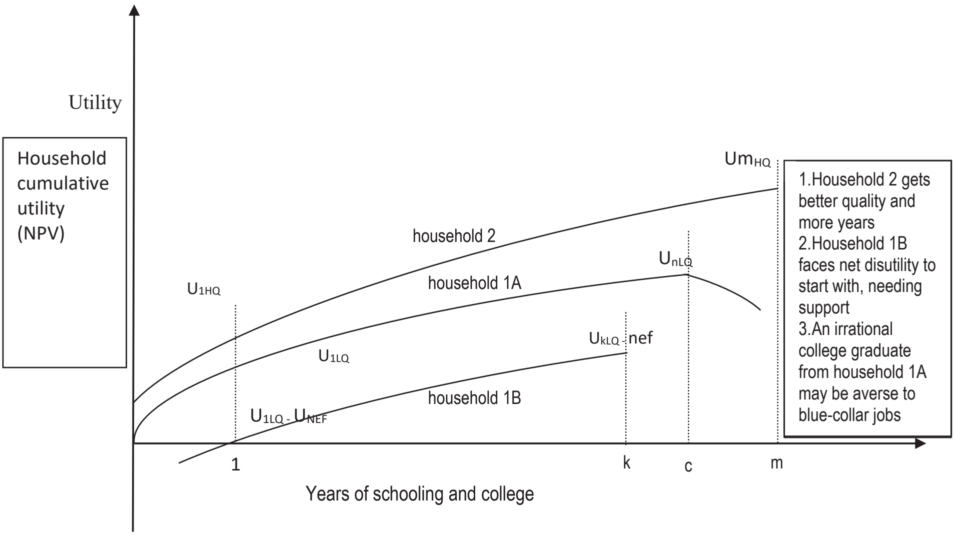
Quantity Demanded		Quantity Supplied
OA	**met** by High quality	no high quality supply beyond OA (so non-market clearing)
$AH_{oversbd}$	unmet by high quality (oversubscribed)	
AB	**met** by low quality	
BC	unmet by low quality too for want of educational expenses	Low quality supply is OD being sufficient for the cohort. Of it AB is availed.
CD	unmet for want of support in addition to educational expenses	

OD	is the total cohort
Demand Curves	
$D_{HQ}D'_{HQ}$	Demand curve of high quality
$D_{LQ}D'_{LQ}$	Demand curve of low quality
Supply Curves	
$S_{HQ}S'_{HQ}$	Supply curve of high quality
$S_{LQ}S'_{LQ}$	Supply curve of low quality

Source: Authors' own depiction.

Note: To ensure that 'no one is left behind' the children represented by BC and CD need to be located, motivated and covered.

Figure 3: Household Cumulative Utility from Years of Schooling and College Education of a Member



Source: Authors' own depiction.

Notes: 1. Years of schooling and college education used as proxy to education. Cumulative utility $U(nQ_i, p_n)$ as a function of cumulative years of schooling 'n', quality level high and low and cumulative price p_n.
2. In the lowest curve the non-economic factors (NEF) are also subtracted.

more granular and rooted in ground realities, as it shows cumulative utility from low quality education, net of cumulative price as well as non-economic factors (NEF) converted into monetary terms, therefore being below the low quality curve. Notably, as shown in the middle curve, after a certain age, an additional block of years of education may rather be utility diminishing due to an irrational behaviour. For instance, a student on acquiring a graduation degree, without any hands-on skill, may neither be employable for want of a skill, nor willing to undertake any blue-collar job.

7.4 Education and 'Star Performers'

Next we carry out an SDG 4 experiment on the scores of students in an assumed small town having two sub-groups, the first in a low quality school having 80 students in two sections, and the second sub-group in a high quality school having 20 students in one section.

Case A: No 'Star Performers':

To start with we assume that the adverse effect of circumstances leads to none of the students in the low quality school scoring more than even the lowest scorer in the high quality school. Thus, there is no overlap of scores or no 'Star Performer'. For simplicity, one can set the scores S_{LowQ} in the low quality school arranged in the non-decreasing order as 0, 35, 41, 41, 42, 42, 43, 43 … 78, 78, 79, 79, where the first two students are facing the most adverse circumstances and next each score is higher by unity which is achieved by two students, forming 39 pairs of such scores. Now, let the scores of the 20 students in the high quality school be 81, 82, 83 ….99 and 100 in the ascending order. This is a case of no overlaps and no 'Star Performers', a condition that would be relaxed next.

Case B: 'Star Performers:

As the focus of our analysis is on how the inequality can reduce between the sub-groups, we use the Gini Coefficient, and for this purpose, we ascertain its 'in-between' component between the two sub-groups. Accordingly, we assume next that five leading students from the low quality school improve their scores from 77, 78, 78, 79 and 79 to 81, 82, 83, 84 and 85, while the two most lagging students scoring 0 and 35, improve their scores to 50. The decomposition analysis is given in **Table 6** which brings out that the Gini coefficients based on both the Traditional as well as the Generic Distribution Revealed Inequity Augmented Lorenz curve, fall by around its one-seventh

Table 6 : Education Scores in Low and High Quality Schools

	No Star Performer (Case A)		Min 50 marks and five Star Performers (Case B)		Remarks
		Per cent of Gini Coefficient (approx.)		Per cent of Gini Coefficient (approx.)	
N	100		100		
p	80		80		
q	20		20		
AM	65.3		67.05		
$AM_{ScoreLowQ}$	59.0		61.19		
$AM_{ScoreHighQ.}$	90.5		90.5		
Gini 'within' LowQ	0.070	47.0	0.056	43.6	
Gini 'within HighQ.	0.002	1.3	0.002	1.5	was 0.002 and is now the same 0.002
Gini coefficient 'in-between'	0.077	51.7	0.070	54.9	falls absolutely but rises as per cent
Totals	**0.149**	**100.00**	**0.128**	**100.00**	

Continued...

Continued...

			14.6 per cent i.e. about 1/7th fall		
	Median 64.5		Median 64.5		
	r_{scores} 0.64		r'_{scores} 0.68		Inequity comes down
	s_{scores} 0.36		s'_{scores} 0.32		
Gini coefficient of the distribution revealed inequity augmented Lorenz curve	0.198		0.168		from case A to B a fall of 14.9 per cent or about one-seventh

Source: Authors' own computations.

8 Aspects of Equity Aimed for and Affirmative Actions

8.1 Towards Equity and an Equitous Lottery experiment:

The education enhances academic knowledge levels in an almost continuous manner tested in terms of discrete cardinal scores or ordinal grades at specified time intervals like the end of the academic year or semester.

Notably, unlike incomes, the resultant academic attainments are not modified through any inter-student progressive/ regressive transfers; but through academic gains available to each decile. Moreover, by making a given additional effort a student in the lower academic decile level stands a chance to improve more, compared to a student in a higher academic decile.

The efficacy of an education framework can be adjudged through how the lowest academic deciles are raised towards a minimal achievement which can be say, 50 per cent scores or even beyond. Certainly, the focus is not limited to how the students in top deciles fare in improving scores, though the underlying idea is not to obstruct their efforts in any way. In reality some of them might have started from the lowest academic deciles. Similarly, the issue of equity in nutrition can be conceived of.

Ferreira *et al.* argue[55] that individuals facing the same circumstances can be categorised as a type (a row in the Table), and those making the same effort as a tranche (a column in the Table). Taking it ahead, in **Table 7 we analyse** a generic case for a student on her score s_{ij} based on circumstances 'i' faced by her, and efforts 'j' made by her, as an n*m matrix. Here, the selection of the number of rows and columns would entail a tradeoff between the choice of larger numbers (so that there are no wide jumps), and smaller numbers (to keep the matrix simple enough). One can of course perceive the classical problem of how to disentangle the effects of circumstances and efforts.

Table 7: Matrix of Circumstances Efforts: Resultant Stochastic Outcome Scores

	e_1	e_2	e_3	…	e_m
c_1	s_{11}	s_{12}	s_{13}	…	s_{1m}
c_2	s_{21}	s_{22}	s_{23}	…	s_{2m}
c_3	s_{31}	s_{32}	s_{33}	…	s_{3m}
…	…	…	…	…	…
c_n	s_{n1}	s_{n2}	s_{n3}	…	s_{nm}

Source: Authors' compilation.

Analysis of inherent issues indicates that firstly, equal income for equal effort (Barry) is *post facto* and so applicable only when one gets a job or starts earning by establishing self-employment. However, the larger real-life issue remains as to how one gets a job or becomes self-employed.

Secondly, the Rawlsian veil of ignorance[56] is not applicable to a student having full knowledge that lower his performance higher the compensation, as he may suppress, especially the self-reported information like his family getting him private tuition or buying quality books and magazines, thus not facing some adverse circumstances on the aspect of parental effort.

Thirdly, the broad aim of the educational framework should be to promote extra effort, a move towards poverty sensitising (PS) equity. There is a need to realise that the tested efforts and abilities may also reflect circumstances, like private tuitions afforded by a 'rich' household for entrance exams may diverge the outcome scores.

In modern economics, the overlapping generations model is termed as the second major workhorse, after the neoclassical growth model (Fernandez-Villaverde, 2021).[57] It indicates that a generation maximises utility over say, two periods (each utility being positive increasing and strictly concave), for which it discounts the utility of the second period and adds it to the full utility of the current period.

The issue of school choice emanates from the scarcity of seats against a higher demand. In this endeavour, one pathway could be to first give high quality school seats to CWSN and orphans, or they are admitted to high quality residential schools providing free boarding and lodging borne by the state, and facility to periodically visit families, if any. Moreover, in a residential school, each teacher may voluntarily adopt say, two to four orphans, to establish life-long family bonds. Thereafter, the balance seats in high quality schools, residential schools, can be allotted through a lottery, which is centralized, through a credible computer software, and no child bagging a seat in a 'low' quality school can swap it with a student bagging a seat in a 'high' quality school. However, the candidates not bagging a seat in lottery, for no fault of theirs, should not remain in the low quality schools till the completion of their schooling (the probability of becoming a drop out would also be higher for such students as the schooling would be relatively less worth).

A way out is that the first lottery is only for the first two years, followed by the next lotteries for admission to say, class three, fifth and seventh. To start with based on availability of high-quality seats we assume a fifty per cent chance of 'success' in each lottery. We are aware that a toss of coin repeated five times gives a probability of (1/32) for the five continuous 'failures' (in the terminology of probability theory, a label that can also be appropriately modified for a school choice). Next, drawing upon probability theory authors conducted a theoretical experiment that a child (whether from a 'rich' or a 'poor' household) 'failing' on the preceding lottery is given an enhanced 3/4 chance of 'success' and thus a diminished 1/4 chance of 'failure' in the next lottery; resultantly the child 'successful' in the preceding lottery has 1/4 probability of 'success' and 3/4 of 'failure'. It is suggested that to avoid the drudgery of minting a biased coin, after the first single fair-coin toss, two fair coins are tossed simultaneously in each subsequent lottery. Further an equitous rule be applied that,

"In case of the preceding 'failure', to be declared 'successful' on opting say, 'head' on bagging '<u>at least one head</u>' i.e. HH or HT or TH thus having a 3/4 probability of 'success', and only 1/4 (TT) of 'failure'. Similarly, on opting say, 'tail', she should be declared successful on bagging '<u>at least one tai</u>l' i.e. TT, or TH or HT, thus having a 3/4 probability of 'success', and only 1/4 (HH) of 'failure'."

As a result after five lotteries, the chance of five consecutive 'failures' now reduces to (1/2)*(1/4)*(1/4)*(1/4)*(1/4) or 1/512 against 1/32. As per local needs such experiments may be considered and credibly implemented, in conjunction with maintaining the Rawlsian veil of ignorance.

The probability distribution in the equitous tossing thus accounts past 'failure' or 'success', no longer rendering different lottery events as independent. It converges the probabilities of 5, 4, 3, 2, 1 and 0 'failures' out of five lotteries (second onwards using two coins) as 1/512 times of 1, 33, 222, 222, 33 and 1; against the traditional probabilities for independent events (converted to the same denominator for comparison) as 1/512 times of 16, 80, 160, 160, 80 and 16. One can visualise convergence, as the extreme chances of a student having five 'failures' or 'successes' is strikingly diminished to 1/512, and even of four 'failures' or 'successes' substantially diminished to 33/512, with resultant substantial increases to 222/ 512 each in the cases of three or two 'failures'.

As a further corrective measure all the students who do not clear the first four lotteries in a row, being only 1/128 or less than one percent, may be given admission to quality schools without being subject to the fifth lottery, thus eventually not leaving any children having never attended a high quality school. In practice much larger addition of high quality seats would be a preferred policy action, reducing the probability of not bagging a high quality seat.

8.2 Affirmative Actions as Measurable Choices for Equity

Now we look into the basic values of fairness and justice and how equity can be aimed in many overlapping ways for household incomes through various affirmative actions as covered in the ensuing cases, followed by its gist in **Table 8.**

Case 1: equi-proportional rise

Each income rises by the same proportion and thus enhancing absolute differences. As absolute differences and AM rise in the same proportion, it results in the Gini coefficient remaining unchanged. Therefore, when it is stated that the Gini coefficient of a country has remained unchanged, it is not necessarily correct to say that the income inequality remains unchanged; as it is a case of divergence in absolute incomes, though relative incomes are unchanged. Therefore, deeper policy interventions should aim at reducing gaps between incomes, and a consequential fall in the Gini coefficient.

Case 2: equal absolute rise

Each income rises by the same absolute amount leading to unchanged absolute differences, but as AM rises, the new Gini coefficient is lower being (old AM/ new AM) fraction of the initial one, indicative of convergence.

Algebraically, if upon an intervention, each term and thus AM increase by AM times ω, where ω (omega) is real and positive, the new arithmetic mean becomes (AM + AM*ω) or AM*(1+ω). Resultantly, the initial Gini coefficient G_0 becomes $G_f = G_0 * \{1/(1+\omega)\}$, or converges by a factor of $(1+\omega)$, an inequality reducing change seen from the lens of Gini coefficient, though absolute income gaps persist.

By varying ω, the partial derivative of G_f can be written as:

$(\partial G_f / \partial\omega) = (-) G_0 * \{1/(1+\omega)^2\}$ or $(-)G_0 * (G_f / G_0)^2$. This implies that by varying ω the rate of fall of the Gini coefficient increases in proportion to the square of its new changing albeit smaller value.

Case 3: equal marginal utility of income for representative 'poor' and 'rich'

This is quite a deeper choice to delve into, as to ascertain marginal utilities (MUs), the shapes of utility curves of both 'rich' and 'poor' would be required. On attaining equality of MUs the Gini coefficient may or may not fall. There can be a situation when curves of both - a representative 'poor' and a representative 'rich'- are identical, and concave to the income axis, utility remaining positive, while initially both representatives being at different points on the curve. In such a situation, marginal utilities can become equal only when incomes become so.

Case 4 to remove financial poverty

Policies can be set to fill the gap for poor between incomes and the financial poverty line. This is akin to filling up the poverty gap or the first moment. the Gini coefficient would fall, and progressive taxation to generate funds would make it fall further.

Case 5 to remove multidimensional poverty (MDP)

The MDP cutoff level is currently set as having more than one-third of the weightage of all deprivations. In fact, in the medium and long-term the deprivation cut off itself would need to be reduced below one-third, say to one-fifth, which in the current MDP literature is termed as the deprivation level indicative of being 'vulnerable' (ranging from one-fifth to one-third). Moreover, the deprivation indicators can be tightened along the journey to attain SDGs by 2030 through a Proactive Multidimensional

Poverty Tracker (PROMPT) (Anand and Kumar 2021).[58] This case of removal of MDP, would lead to a fall in the Gini coefficient, and a progressive taxation would further contribute towards its fall.

Case 6: equal public welfare spending

This is akin to Universal Basic Income (UBI), aimed at the welfare of everyone without the need to apply means-testing. In the process the amount made available to each person may become too small or all the ongoing welfare schemes may need to be closed. An alternative termed as 'Universal Programme with Likelihood Inbuilt for Fair Targeting (UPLIFT)' (Anand and Kumar)[59] can help in maintaining a significant provision for all those who opt to avail, keeping the intervention universal in nature, as an example an adult education facilities. It would lower the value of the Gini coefficient, which is further reduced by progressive taxation to fund it.

Case 7 equal marginal utility of the public spending

This can help in reducing the gap attributable to circumstances, as to start with a 'poor' would have a higher marginal utility. This should not be limited to short-term low hanging fruits but trigger these to pave the way for long-term corrective actions. The Gini coefficient would fall, and diminish further due to a progressive taxation to fund it.

Case 8: ceiling on the ratio between the highest and the lowest incomes:

This is a measure necessitating appropriate policies especially fiscal, monetary and distributive ones to ensure the ceiling on the ratio. The desired ceiling ratio may be denoted as Φ, a positive number (say, 10). This in turn can have innumerable scenarios across the distribution. The two extreme income scenarios are being covered next, having either a sole 'rich' or a sole 'poor'.

Scenario 8.1 When the income x_{rich} of the sole 'rich' is Φ times of x_{poor} the income of each of the (N-1) 'poor', with the average income being AM, and $x_{rich} = \Phi * x_{poor}$ the following equation would hold:

$$(N-1)* x_{poor} + 1*\Phi*x_{poor} = N*AM,$$

or, $x_{poor} = (N*AM) / (N+ \Phi -1)$, and $x_{rich} = (\Phi*N*AM) / (N+ \Phi -1)$. In the double summation, the (N-1)*(N-1) pairs of 'poor', and the self-pair of the sole 'rich' yield zeros. The 2*(N-1) cross terms yield 2*(N-1)*(x_{rich} - x_{poor}), leading to the Gini coefficient as, {2*(N-1)* (Φ-1)*N*AM} / {(N+ Φ -1)*AM*2*N*N},

or, {(N-1)*(Φ-1)} / {N*(N+ Φ -1)},

So for N=100 and Φ=10, the Gini coefficient becomes (99*9) / {(100* (109)}, or approximately 0.08. Moreover, for N > > Φ > 1, it tends to (Φ-1) / (N+ Φ -1), or (Φ-1) / (N).

Scenario 8.2 When the income x_{rich} of each of the (N-1) 'rich' is Φ times of x_{poor} the income of sole 'poor'. With the average income being AM, and $x_{rich} = \Phi * x_{poor}$ the following equation would now hold:

$$(N-1)*\Phi*x_{poor} + 1*x_{poor} = N*AM,$$

or, x_{poor} = (N*AM) / [{(N-1)*Φ} +1)], and x_{rich} = (Φ*N*AM) / {(N-1)*Φ +1)}. In the double summation, the (N-1)*(N-1) pairs of 'rich', and the self-pair of the sole 'poor' now yield zeros. The 2*(N-1) cross terms yield 2*(N-1)* (x_{rich} - x_{poor}), leading to the Gini coefficient as,

{2*(N-1)*(Φ-1)*N*AM} / [{(N-1)*Φ +1)}*AM*2*N*N},

or, {(N-1)*(Φ-1)} / [(N)*{(N-1)*Φ +1)}],

So now for N=100 and Φ=10, the Gini coefficient becomes (99*9) / {(100*(991)}, or approximately 0.009, which for N > > Φ > 1, tends to (Φ-1) / {(N-1)*Φ +1)}, or (Φ-1) / (Φ*N).

Importantly, under both scenarios, though for N being large the Gini coefficients differ, still it is not very high. The underlying reason is that in both scenarios, as many as (N-1) out of the N terms have identical values, and many other scenarios would be more divergent.

Case 9: towards equality of incomes earned by efforts with adverse circumstances duly compensated

Here, we visualise equality of the component of incomes earned by efforts, under the assumption that all adverse circumstances have been duly compensated. Further, incomes from a differential in wealth are not included, and it is assumed that fiscal policies would reduce the wealth gap through a progressive estate duty or inheritance tax. In the case of education, the aim would be equalization of scores and eventually skilling. In the case of nutrition, in the long-term it would be to have a society in which AM of the weight of girls is no longer below that of boys. Obviously, the Gini coefficient would tend to zero, on the component otherwise affected by circumstances.

The cases 1 to 9 are summarized together in **Table 8** for easier comparison.

Table 8: Various Cases of Affirmative Actions Towards Equity in Incomes

Case	Equity aimed for	Effect on Gini coefficient	Additional fall due to progressive taxation to fund	Remarks
1.	equi-proportional rise	unchanged	not applicable	absolute differences increase
2.	equal absolute rise	falls	yes	absolute differences remain unchanged
3.	equal marginal utility of income for representative 'poor' and 'rich'	may or may not fall	not necessarily	MUs depend upon the shape of the utility curves
4.	to remove financial poverty	falls	yes	to meet minimal financial needs

Continued...

Continued...

5.	to remove multidimensional poverty (MDP)	falls	yes	The MDP cutoff level is currently set as one-third of the weightage of all deprivations, which should also be reduced
6.	equal public welfare spending	falls	yes	It is akin to UBI
7.	equal marginal utility of the public spending	falls	yes	short-term low hanging fruits shouldn't crowd out log-term structural corrective actions but trigger these to pave the way for long-term corrective actions
8.	ceiling on ratio between the highest and the lowest incomes	falls	yes	assuming that while these two incomes are taken care of, the intra-range distribution doesn't become too unequal
9.	towards equality of incomes earned by efforts with adverse circumstances duly compensated	tends to zero	yes	can be conceived to the extent of compensating for adverse circumstances, and also for the retarding effect of circumstance on effort

Source: Authors' own composition.

9 Conclusions

Childhood challenges of undernutrition, low quality school education and household's multidimensional poverty are deeply intertwined for the current generation; and in the temporal manner for the next generation and beyond. Inclusive policies that currently promote adequate nutrition, quality education (including digital education) and household employability can enhance incomes and prove to be the *sine qua non* of early childhood care, development and progress. These are the key factors in evolving any affirmative policy actions towards childhood, treating it as a global pure public good. Besides, clearance of stipulated physical standards and awareness to later on acquire skilling leading to employability are also essential foundations of childhood care.

These objectives are also part of the SDGs, and in fact within these on malnutrition, there are intermittent target levels to be achieved by 2025. Notably, the triple burden of malnutrition encompassing undernutrition, overweight and micronutrient deficiency remains a critical childhood challenge. In it, undernutrition manifests as childhood stunting, wasting and underweight, its risk being the highest for the children simultaneously facing all these three anthropometric failures.

It is a crucial fact that any assessment of anthropometric failures say, at the age of five, pertains only to such children in the cohort who could survive till this age. Therefore, it overlooks the children who died before reaching this age. So, two samples of children of age five, showing similar values of undernutrition indicators of stunting, wasting and overweight; may still significantly differ in actual status of cohorts due to differential mortality rates till age five.

The most critical challenge is to answer 'equity of what', though equality also needs a defined space, like not being underweight or overweight still relatively easier to define as an objective for children across sub-groups. An analysis of how equity can be perceived in different ways necessitating affirmative actions is also carried out, along with the likely changes in the value of Gini coefficient.

Achievements in life are linked to circumstances, efforts and their two-way interaction terms, while normatively efforts need to be rewarded and adverse circumstances need to be compensated. To conceive a policy addressing this concern the major challenge is to disentangle the effects of circumstances from that of efforts. On zero hunger the term efforts should not be confined to affordability to buy food items. It includes opting for nutrition-rich food, and avoiding obesity and smoking. The mental aspect of health and peace further necessitate calmness and being balanced as it makes one more productive for the society.

On the aspect of education, the early childhood education is the critical pathway to better knowledge and eventual skilling leading to life-long opportunities. However, education remains scarce when it comes to high quality, towards which the state has a greater responsibility to shoulder. In education, the objective of an affirmative action is expected to give children from disadvantaged groups the ability to get admission to a high quality school. Unfortunately, for many households retaining the child at home is the best option and the opportunity cost being to altogether miss schooling. This is in contrast to the societal best option to promote universal schooling.

Drilling down on the quality issue, vertical product differentiation is analysed to capture the choice between two levels of quality of a product in general and especially in the case of uniqueness of school education. Besides, two diagrammatic manifestations are evolved for 'Early Childhood Education: Market Demand and Supply for Dual Quality Vertical Differentiation for a Cohort of Children' covering out of school children also; and 'Household Cumulative Utility from Years of Schooling and College Education of a Member'.

It is known by definition that the Gini coefficient in essence captures (normalised) double summation of absolute differences, without assigning any extra weightage to terms below arithmetic mean (AM) compared to the terms above AM. Therefore, it is an indicator of inequality *per se*, but not of inequity. Still, it has a number of properties shown in this paper bringing out its usefulness in capturing its 'in-between' sub-groups component.

A fair tax/ subsidy rule is covered based on proportional tax/ subsidy on the distance of income from the AM; its special feature being that it distributes all the taxes collected as subsidies and preserves ranks. Next, contributing to the literature a distribution revealed inequity parameter termed 'Generic Inequity Augmented Lorenz Curve' is computed by using the ratio between the sum of terms below the median to the sum of terms above it, while assigning half of any term exactly equal to the median to both sides.

As by definition the number of terms below and above the median is equal, the ratio called 'r' is invariably less than or equal to one, the case of equality occurring only when each term is equal, rendering no inequality. Incidentally, in the special case of an SDG 10.1 experiment with incomes uniformaly increasing, the AM and the median can be interchangeably used. From the distribution revealed ratio 'r', one can compute a parameter say, 's', called as the inequity gap parameter defined as (1-r), which is used as 't' the tax/ subsidy rate for reduction of inequity under the fair tax and subsidy rule. Using this tax/ subsidy rate for each term x_i a new term {AM – t*(AM-x_i)} is computed, called v_i fraction of x_i, converging the terms. Thereafter, the division (x_i/v_i) results in a divergent distribution helping in this paper to evolve a generic inequity augmented Lorenz curve.

In the literature on inequality indices, the additive decomposition of any index remains an important feature. A related issue is of the overlap i.e. if a member of the disadvantaged sub-group, say a rural household attains income higher than that of a member of the advantaged sub-group, here an urban household, then such absolute difference between the two members is termed as an overlap. In this paper, such a better performing member is called as a 'star performer'. Further, also contributing to the literature a new index called the 'Star Performance Index' i.e. SP Index is also evolved as the ratio of the sum of all overlap terms among the 'in-between' terms to the double absolute summation of the 'in-between' terms. This is also termed as Poverty Sensitising Index 5 (PS5), in continuation of other PS indices evolved earlier and mentioned in this paper. A salient feature of the SP Index proved in the paper is that its value varies between zero and unity. Further, in this paper, the overlap case is also analysed extensively and algebraically linked to carry out additive decomposition of the Gini coefficient.

10 Recommendations

i. The early childhood needs to be accorded the formal status of a global public good. Further, this public good should be made as close as possible to a global pure public good, by making it largely non-rival and non-excludable, through much higher resource allocation.

ii. Attainment of SDGs targets 2.2 and 4.2, as focused in this paper and having bearing on most SDGs, necessitates enhanced allocations of domestic resources as well as deeper international collaborations.

iii. A broader policy focus should be to minimise the adverse effects of circumstances, rather than to first let these create inequities. The current cyclic syndrome can be expressed as: persistent iniquitous circumstances – increasing inequality - affirmative action - persistent iniquitous circumstances. It should ideally be modified to the growth trajectory with compensated iniquitous circumstances and reduced inequalities harnessing the inherent potential for development and prosperity, towards which early childhood is the bedrock.

iv. Seen from the lens of the circumstances, stunting can be avoided with the provision of nutrition care centres, like the *Anganwadi* centres under the *Saksham* & Poshan 2.0 programme in India. These need to be funded by the State and various centres run by State/ private sector/ civil society and encompass nutrition, pre-schooling and related early child interventions like education and awareness on nutrition and health, immunisation, health check-ups, referral services; covering along with adolescent girls, pregnant women and lactating mothers. These can be suitably modified in other countries to meet the local needs.

v. If a mother desires to opt for paid work, crèche facilities for the job or a cluster of jobs should be provided. For a disabled child, medical treatment and aid needs to be ensured to diminish her handicap, and the best care meant for a child with special needs (CWSN) should be imparted. It is further recommended that such children should be main-streamed by facilitating to attend the same classroom as for regular students.

vi. Focus on qualitative aspects needs to be enhanced to attain better efficiencies and synergy. Going beyond calorific needs it is essential to focus on nutrition-rich items safe from harmful bacteria and virus, and providing better absorption in the body. Moreover, better quality of education can be achieved by improving cognitive skills, promoting participative games and sports, evaluations based on better outcomes; and supported by forward looking, inclusive and values supporting pedagogies.

vii. Considering short-term inadequacy of seats in high quality schools, firstly CWSN and orphans may be covered through free high quality schools and residential schools, reimbursed by the state. Remaining seats may be allotted through an equitous lottery as evolved in section 8, further ensuring at least a couple of years of high quality education for each student so that 'no one is left behind'. Simultaneously the number of high quality seats should be increased through additional shifts like evening shifts, improvement of quality of schools and establishment of new better quality schools. These are not purely acts of benevolence, but the society gets paid back through higher productivity, and in narrow financial terms through the higher present value of the future stream of revenues.

viii. The policy space needs to address the issue of zero schooling demand even for zero price (or negative price i.e. subsidy) for the most deprived sections, as for them costs including the opportunity cost of sending a child to the school, necessitate to partly subsidise non-educational costs. It should be visualised that for some households incomes may be zero, dis-savings not possible for want of any savings, borrowings may not materialise; and so little safety nets or social assistance even though not automatically available, may be the only form of receipts. Children from such households should also be identified and to the extent possible provided free boarding and lodging.

ix. It is not recommended to give automatic admission to a better quality school on the grounds of being the sibling of a student already studying in such a school. If objective of parents is to get admission in the same school, when the younger sibling bags a seat only in a low quality school, they may shift the elder child to the low quality school. Of course, whether from 'rich' or 'poor' households, fair mechanisms should be in place to ensure that such children do not remain continuously deprived of high quality education over the years.

x. Proximity of residence to school acts adversely for disadvantaged children, as their households are situated in localities away from where affluent people live and benefit from a higher probability for high quality schools to establish in affluent areas. It may better be avoided as an admission criterion, in order to help the disadvantaged.

xi. To mainstream children with special needs (CWSN) and sensitise others better, one modality could be that some institutes are headed by persons who overcame similar circumstances and are selected on academic merits, to help bridge the divide. Regarding sensitisation, for an instance if able children can learn basics in Braille it would help them to empathise better, besides say, ability to use it in dark places.

xii. There is a need to credibly prove the importance of academic achievements, skilling and physical fitness in the job market. The jobs of repetitive nature can be linked to minimal laid fitness standards and awarded on the basis of academic and skilling grades achieved during schooling. Thus, a signal would go on the utility of passing exams with higher grades, so essential to utilise younger formative years aligned towards better educational outcomes. Life-long improvement of academic and skilling grades, through interventions like adult education and skilling should also be utilised as instruments of the human capital enhancement policy.

xiii. Mother's education used as an input variable in many studies is limited to the binary choice of her being literate or otherwise. Studies need to be graduated to capture quality oriented indicators like the level and grades attained by mother in education, her skill level, employability. Of course, if compared to fathers' education, mothers' education is found statistically more significant, and thus to avoid the problem of multicollinearity, only mother's education may be included in a regression analysis.

xiv. If the Gini coefficient of a country remains unchanged, it is not necessarily correct that the income inequality remains unchanged; as it is a case of divergence in absolute incomes, though either relative incomes may remain unchanged, or inequalities change in different portions of two Lorenz curves, while the non-overlapping lens regions remaining equal. Therefore, deeper policy interventions should be undertaken aiming at reducing gaps between absolute incomes, to overcome the direct adverse effects of circumstances and indirect effects of circumstances on efforts. A better test of such policies should be to achieve a commensurate fall in the Gini coefficient.

xv. The idea of generic inequity augmented Lorenz curve revealed by the given distribution, contributed to literature in this paper needs to be promoted to sensitise on the inherent inequity not captured in the traditional Lorenz curve.

xvi. Similarly the ideas of 'star performer' and the 'star performance index' also contributed to literature in this paper, being inherently equitous in nature, should be promoted to help the disadvantaged sub-groups.

xvii. In real life situations, as the nutrition or education related parameters are not exactly normally distributed, techniques like the LMS method, where these letters denote Lambda, Mu and Sigma (LMS), can be utilised. These help to modify the given actual distribution towards a normal distribution. Especially for handling the lowest and the highest percentiles, these corrective transformations should be promoted to construct smooth curves closer to reality, for undertaking improved applications.

xviii. There is a need for involving all stakeholders in the evolution of policies that treat early childhood as a global pure public good, supported by tracking each child through the latest technological breakthroughs. Periodic policy assessments should be undertaken to appraise progress across sub-groups differentiated by circumstances, to revisit and improve upon policies.

Endnotes

[1] https://unstats.un.org/sdgs/report/2023/The-Sustainable-Development-Goals-Report-2023.pdf pp 14 and 20

[2] UNESCO. 2022. Tashkent Declaration and Commitments to Action for Transforming Early Childhood Care and Education, Tashkent.

[3] Nandy, S., Irving, M., Gordon, D., Subramanian, S.V. and Smith, G.D. 2005. Poverty, child undernutrition and morbidity: new evidence from India. *Bulletin of the World Health Organization*, *83*, pp.210-216.

[4] Nandy, S. and Svedberg, P. 2012. The Composite Index of Anthropometric Failure (CIAF): An alternative indicator for malnutrition in young children. In *Handbook of anthropometry: Physical measures of human form in health and disease* (pp. 127-137). New York, NY: Springer New York. DOI https://doi.org/10.1007/978-1-4419-1788-1_6,

[5] Pomati, M. and Nandy, S. 2020. Assessing progress towards SDG2: Trends and patterns of multiple malnutrition in young children under 5 in West and Central Africa. *Child Indicators Research*, *13*(5), pp.1847-1873. https://doi.org/10.1007/s12187-019-09671-1.

[6] Maharana, Barsharani and Ladusingh, Laishram. 2022. What Explains the Decline in Infant Mortality in Demographically Underdeveloped States in India?. The Case of Uttar Pradesh and Madhya Pradesh. *Economic and Political Weekly*, Vol. LVII No. 47, 19th November.

[7] Culyer, A. 2015. Efficiency, equity and equality in health and health care. Discussion Paper. CHE Research Paper. Centre for Health Economics, University of York , York, UK, 14p.

[8] Chaturvedi, Sachin, Saha, Sabyasachi and Nair, Arun S. 2019.Social Stock Exchange for Social Enterprises and Social Incubators: An Exploratory Study for India, New Delhi, RIS, *Discussion Paper 243*, 23p.

[9] Ashenfelter, O. and Rouse, C. 1996. Income, Schooling, and Ability: Evidence from a New Sample of Twins (No. 744). Princeton University, Department of Economics, Industrial Relations Section.

[10] Michael Spence, Spence, Michael. 1974. Competitive and Optimal Responses to Signals: An Analysis of Efficiency and Distribution. *Journal of Economic Theory*, 7(3), pp 296-332.

[11] Amadeo, K. and Boyle, M.J. 2021. What Is Educational Equity and Why Does It Matter? Why Equity Is Better Than Equality for the Economy https://www.thebalancemoney.com/equity-in-education-4164737

[12] US, Department of Education. 2022 Agency Equity Plan Related to Executive Order 13985, 14th April. https://www2.ed.gov/documents/equity/2022-equity-plan.pdf

[13] Burgess, Simon; Greaves, Ellen and Vignoles, Anna. 2020. School Places: A Fair Choice?: School choice, inequality and options for reform of school admissions in England. pp 7 and 8, The Sutton Trust. https://www.suttontrust.com › uploads › 2020/02 last accessed on 30th January 2023.

[14] OECD. 2008. Ten steps to Equity in Education. *Policy Brief*, pp 8. https://www.oecd.org/education/school/39989494.pdf last accessed 30th January 2023.

[15] Castelli, Luciana, Ragazzia, Serena and Crescentin, Alberto. 2012. Equity in education: a general overview, International Conference on Education and Educational Psychology (ICEEPSY 2012).

[16] Piketty, Thomas. 2014. Capital in the Twenty-First Century. Cambridge Massachusetts, The Belknap Press of Harvard University Press.

[17] Ibid. pp. 305.

[18] Munoz-Garcia, Felix. Vertical differentiation and natural monopoly, Washington State University, USA.

[19] Dunaway, Eric. 'Vertical Product Differentiation, Washington State University'.

[20] Hafalir, I.E., Yenmez, M.B. and Yildirim, M.A. 2013. Effective affirmative action in school choice. *Theoretical Economics*, *8*(2), pp.325-363.

[21] Wouters, A., Croiset, G. and Kusurkar, R.A. 2018. Selection and lottery in medical school admissions: who gains and who loses?. *MedEdPublish*, *7*, p.271.

[22] Antoninis, Manos; Delprato, Marcos; Benavot, Aaron. 2016. Inequality in education: the challenge of measurement. University of Sussex. Chapter. https://hdl.handle.net/10779/uos.23448668.v1

[23] Okun, Arthur. 1975. Equality and Efficiency: The Big Tradeoff. Oxford & IBH Publishing House, pp. 43.

[24] Ibid. pp. 91.

[25] Ibid. pp 101.

[26] Tyson, Laura and Spence, Michael. 2017. Exploring the Effects of Technology on Income and Wealth Inequality, Chapter Eight In *After Piketty- The Agenda for Economics and Inequality, edited by Boushey, Heather; Delong, J. Bradford and Steinbaum*, Marshall. Harvard University Press, pp. 204.

[27] Shields, L. 2023. Equality of Educational Opportunity. Stanford University on Plato, California, US pp 1.

[28] FAO. 2006. Inequality Analysis - The Gini Index, EASYPol Module 040. http://www.fao.org/3/a-am352e.pdf

[29] FAO. 2006. Policy impacts on inequality: decomposition of income inequality by subgroups by L.G. Bellù and P. Liberati. *EASYPol Module*, *52*.

[30] Fairlie, R.W. 1999. The absence of the African-American owned business: An analysis of the dynamics of self-employment. *Journal of labor Economics*, *17*(1), pp.80-108.

[31] Jusot, F., Tubeuf, S. and Trannoy, A. 2010. Effort or circumstances: does the correlation matter for inequality of opportunity in health?. *IRDES Working Papers* DT33, 33p.

[32] Barry B. 2005. Why Social Justice Matters. Cambridge, Polity Press.

[33] Roemer J. 1998. Equality of Opportunity. Harvard University Press, Cambridge.

[34] Swift A. 2005. Justice, Luck and the Family: Normative Aspects of the Intergenerational Transmission of Economic Status *In Bowles Ss., Gintis H., Osborne-Groves M., eds. Unequal Chances: Family Background and Economic Success.* Princeton University Press. Princeton; 256-76.

[35] Amadeo, K. and Boyle, M.J. 2021. What Is Educational Equity and Why Does It Matter? Why Equity Is Better Than Equality for the Economy. https://www.thebalancemoney.com/kimberly-amadeo-3305455

[36] Sen, Amartya. 1995. Inequality Reexamined. Russell Sage Foundation, Oxford University Press, pp 88 and 132.

[37] Arneson, R.J. 2007. Does social justice matter? Brian Barry's applied political philosophy. *Ethics*, *117*(3), pp.391-412. pp 3. https://doi.org/10.1086/5117322007 BBS.

[38] Besley, Timothy and Kanbur, Ravi. 1990. The Principles of Targeting. World Bank, WPS, 385 pp 11.

[39] Ministry of Consumer Affairs, Food & Public Distribution, Press Information Bureau 10th December 2021, New Delhi.

[40] Named after the French mathematician L'Hopital, and applied once or more till limit of a derivative of a function. may be reached, if limit of the function is indeterminate.

[41] Theil, Henri. 1967. Economics and information theory. North-Holland, Amsterdam.

[42] Anand, Pramod Kumar and Kumar, Krishna. 2022. SDG Target 10.1 Inequity and Inequalities: Measurement Choices and Building Blocks of Poverty Sensitising Indices. *RIS Discussion Paper # 273*, New Delhi, RIS.

[43] De Battisti, Francesca, Porro, Francesco and Vernizzi, Achille. (2019). 'The Gini Coefficient and the Case of Negative Values'. Electronic Journal of Applied Statistical Analysis (EJASA), Vol. 12, n.1, 2019, 85-107. Available at SSRN: https://ssrn.com/abstract=3610829

[44] Shorrocks, A.F. 1980. The class of additively decomposable inequality measures. *Econometrica: Journal of the Econometric Society*,48(3) pp. 622.

[45] Anand, Pramod Kumar and Kumar, Krishna. 2022. SDG Target 10.1 Inequity and Inequalities: Measurement Choices and Building Blocks of Poverty Sensitising Indices. *RIS Discussion Paper # 273*, New Delhi, RIS.

[46] Atkinson, A. 1970. On the Measurement of Economic Inequality. *Journal of Economic Theory* 2(3) pp 244–263.

[47] Yitzhaki S. 1983. On the Extension of the Gini Index, *International Economic Review*, **24**,pp. 617-628.

[48] Wessa.net free statistics and forecasting software, Wessa P. 2021. Concentration and Inequality (v1.0.2) in Free Statistics Software (v1.2.1), Office for Research Development and Education, URL last accessed on 5th May 2023, http://www.wessa.net/rwasp_concentration.wasp/

[49] Khadilkar, Vaman., Yadav, Sangeeta., Agrawal, KK., Tamboli, Suchit., Banerjee, Monidipa., Cherian, Alice., Goyal, Jagdish P., Khadilkar, Anuradha., Kumaravel, V., Mohan,V., Narayanappa, D., Ray, I., and Yewale, Vijay. 2015. Revised IAP Growth Charts for Height, Weight and Body Mass Index for 5- to 18-year-old Indian Children. Indian Pediatrics, 52: pp. 47-55.

[50] Cole, T. J. and Green, P.J. 1992. Smoothing reference centile curves: the LMS method and penalized likelihood. *Statistics in medicine*, *11*(10), pp.1305-1319.

[51] Shields, L. 2023. Equality of Educational Opportunity, Stanford University on Plato, California, pp. 21.

[52] Stiglitz, Joseph E. 2012. Price of Inequality. W. W. Norton & Company, pp. 267.

[53] Ulbrich, Holley H. (2011), 'Public Finance in Theory and Practice' Routedges, Second editon

[54] Machado, Matilde. Industrial Organization. http://www.eco.uc3m.es/~mmachado/

[55] Ferreira, Francisco and Peragine, Vito. 2015. Equality of Opportunity: Theory and Evidence. *IZA Discussion Papers No. 8994*. Institute of Labor Economics (IZA).

[56] Rawls, John. 1971. A theory of justice. Cambridge, Massachusetts :The Belknap Press of Harvard University Press.

[57] Fernandez-Villaverde. 2021. Jesus, University of Pennsylvania.

[58] Anand, Pramod Kumar and Kumar, Krishna. 2021. Skilling, Fulcrum of a Proactive Multidimensional Poverty Tracker Pathway. *RIS Discussion Paper # 269*, New Delhi, RIS.

[59] Anand, Pramod Kumar and Kumar, Krishna. 2021. Post-pandemic Social Security Agenda: Universalising Developmental Interventions Over Universal Basic Income. *RIS Discussion Paper # 263*, New Delhi, RIS.

RIS Discussion Papers

Available at: http://www.ris.org.in/dicussion-paper

DP#286-2023 *Financial Literacy across Different States of India: An Empirical Analysis by* Priyadarshi Dash & Rahul Ranjan

DP#285-2023 *Promoting Districts as Export Hubs in the Export Policy India's Experiment with The One District One Product Programme* by Pankhuri Gaur

DP#284-2023 Assessing India-Vietnam Maritime Trade: An Empirical Exploration *by Prabir De & Tuhinsubhra Giri*

DP#283-2023 Traditional Medicine in SAARC: A Regional Cooperation Framework *by Namrata Pathak*

DP#282-2023 *Evolving Conceptual Framework for Measuring Wellbeing for Decision and Policy Making by Krishna Kumar and P. K. Anand*

DP#281-2023 *Cost of Remittance: A Priority Area for G20 Member Countries* by Sushil Kumar

DP#280-2023 *Digitalisation for Inclusive Development: An Overview of Digital Strategies and Public Policies in the Europe and India* by Amit Kumar

DP#279-2023 *International Trade and Wage Inequality in India: Does Direction of Trade Matters?* by Pankaj Vashisht

DP#278-2023 *Measuring the SDG Progress in India with Focus on Gap Analysis Approach by Krishna Kumar and P. K. Anand*

DP#277-2023 *Asian Development Bank: Scaling up of Capacity for Enhanced Operations through Combination of Resources* by Prabodh Saxena

DP#276-2023 *G20 Initiatives in Infrastructure Investment & Finance: Progress & Shortfalls by* G. A. Tadas

DP#275-2023 *Potential Role of Academia-Industry Interface for Space Economy: Emerging Policy Options Before India* by Chaitanya Giri

DP#274-2022 *Indo-Pacific Strategies: What do They Entail for India by* Pankaj Vashisht

RIS **A Think-Tank of Developing Countries**

Research and Information System for Developing Countries (RIS) is a New Delhi-based autonomous policy research institute that specialises in issues related to international economic development, trade, investment and technology. RIS is envisioned as a forum for fostering effective policy dialogue and capacity-building among developing countries on global and regional economic issues.

The focus of the work programme of RIS is to promote South-South Cooperation and collaborate with developing countries in multilateral negotiations in various forums. RIS is engaged across inter-governmental processes of several regional economic cooperation initiatives. Through its intensive network of think tanks, RIS seeks to strengthen policy coherence on international economic issues and the development partnership canvas.

For more information about RIS and its work programme, please visit its website: www.ris.org.in

Research shaping the development agenda

RIS@40

RIS

Research and Information System for Developing Countries

विकासशील देशों की अनुसंधान एवं सूचना प्रणाली

Core IV-B, Fourth Floor, India Habitat Centre
Lodhi Road, New Delhi-110 003 India., Tel. 91-11-24682177-80
Fax: 91-11-24682173-74, Email: dgoffice@ris.org.in
Website: http://www.ris.org.in